

On-shell renormalization of neutralino and chargino mass matrices in R -parity violating models - Correlation between LSP decays and neutrino mixing angles revisited

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Abstract

We present an on-shell renormalization scheme for the neutralino and chargino mass matrices for two R -parity violating models, namely bilinear R -parity violation and the $\mu\nu$ SSM with one right-handed neutrino superfield. We discuss in both models how to obtain neutrino masses and mixing angles correctly as well as differences to the existing $\overline{\text{DR}}$ calculations. Both models predict correlations between neutrino mixing angles and ratios of R -parity violating decays such as $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ which can be tested for example at the LHC. We point out that there are cases where one has to include the complete NLO corrections for the decays $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ to predict reliably the correlations between neutrino- and collider-physics. Moreover, we discuss briefly the implementation of these decays in the public program `CNNDecays`.

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1. Introduction

The Minimal Supersymmetric extension of the Standard Model (MSSM) [1] is based on conserved R -parity, where R -parity (R_p) [2] is defined as $R_p = (-1)^{3B+L+2s}$ and originally guaranteed the stability of the proton in supersymmetric models [3]. It implies on the one hand a stable lightest supersymmetric particle (LSP) and predicts on the other hand zero neutrino masses as in the Standard Model (SM). As neutrino oscillation experiments have shown at least two neutrinos have non-zero mass [4] requiring an extension to explain neutrino data¹.

In the SM exist various ways to explain neutrino data, e.g. postulating very heavy particles as in the seesaw mechanism [11, 12] resulting in a unique dimension-5 operator [13] or via loop-effects [12, 14, 15]. Although all mentioned solutions can be supersymmetrized, there exists an entirely supersymmetric possibility to generate Majorana neutrino masses, namely R -parity violation \mathcal{R}_p [16, 17, 18]. Allowing for all possible gauge- and supersymmetric invariant terms within the MSSM particle content, R -parity can be broken by bilinear or trilinear terms [17]. Trilinear models provide a huge number of free parameters, where different attempts tried to reduce them [19]. Spontaneous R -parity violation can be understood as a violation of lepton number by the vacuum expectation value of some singlet field [16, 20, 21]. Bilinear \mathcal{R}_p can be interpreted as the low-energy limit of some spontaneous \mathcal{R}_p model, where the new singlet fields are all decoupled. The bilinear model has only six new \mathcal{R}_p parameters and is thus more predictive in particular in view of collider physics once the parameters have been chosen to explain correctly neutrino data. For an early work on bilinear R -parity violation in the context of a low-energy effective theory we refer to [22].

¹A recent analysis combining the newest precision results from various experiments like KamLAND [5], Super-K [6], SAGE [7], SNO [8] or MINOS [9] can be found in Ref. [10].

The phenomenology of \mathcal{R}_p SUSY has been studied extensively in the past, for reviews see [23, 24, 25]. Neutrino masses have been calculated with trilinear couplings [17, 26] and for pure bilinear models [27, 28, 29, 30, 31] or both [32]. Neutrino angles are not predicted in either schemes, but can be easily fitted to experimental data. In bilinear schemes the requirement to correctly explain neutrino data fixes all \mathcal{R}_p couplings in sufficiently small intervals, so that in some specific final states of the decays of the LSP correlations with neutrino angles appear. This has been shown for a (bino-dominated) neutralino LSP in [33, 34, 35], for charged scalar LSPs in [36], for sneutrino LSPs in [37, 38], and for chargino, gluino and squark LSPs in [37]. Such a tight connection between neutrino physics and LSP decays is lost to some extent in the general trilinear-plus-bilinear case.

The μ -problem of the MSSM [39] is induced by the mass term for the Higgs superfields $\mu \hat{H}_d \hat{H}_u$. For phenomenological reasons the parameter μ must be of the order of the electroweak scale. However, if there is a larger scale in the theory, the natural value of μ should be around this large scale. A solution to this problem can be provided by the Next-to-Minimal SSM (NMSSM) [40, 41], where an additional singlet superfield is introduced. The vacuum expectation value of its scalar component generates an effective μ term as soon as electroweak symmetry is broken. The phenomenology has been discussed for example in [42, 43]. Electroweak mass corrections were discussed in [44], an on-shell scheme for neutralino and chargino masses including the discussion of the two-body decays $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^\pm W^\mp$ at one-loop level was worked out in [45].

The $\mu\nu$ SSM, which was first proposed in [46], uses the same singlet superfield(s) not only to generate the μ term but in addition Dirac mass terms for the left-handed neutrinos. The cubic self-couplings of the singlets in the superpotential explicitly break lepton number and moreover also R_p . Thus, Majorana neutrino masses are generated once electroweak symmetry is broken. One-loop corrections to the neutrino mass matrix have been calculated in [47, 48]. Compared to the NMSSM the $\mu\nu$ SSM with one right-handed neutrino superfield also contains six new \mathcal{R}_p parameters, the same number as going from the MSSM to the bilinear \mathcal{R}_p model as will be discussed in detail later on. This implies that one also obtains correlations between neutrino mixing angles and ratios of LSP decay branching ratios [47, 49]. Moreover, one has unusual decay modes and signatures in the Higgs sector [47, 50].

Although one expects correlations between neutrino mixing angles and final states of the LSP decays at tree level, it is a priori unclear what happens at one-loop level. In particular using one-loop on-shell masses and the one-loop mixing matrix together with the tree level decay widths results in case of the $\mu\nu$ SSM with one right-handed neutrino superfield in an unexpected behavior: The expected correlation does not necessarily show up for a singlino-like LSP. Therefore, we present in this paper a full one-loop calculation for the decays $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ and show how the tree level expectations evolve at full one-loop level. For this aim we extend the on-shell scheme presented in [45] for the decays $\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^\pm W^\mp$ within the MSSM and the NMSSM to the models considered here. Moreover, we briefly discuss the implementation of these decays in `CNNDecays` [51]. Note that in case of the $\mu\nu$ SSM with more than one right-handed neutrino superfield similar problems do not show up. The reason is, that neutrino physics can be fully explained at tree level and thus at the same

level of perturbation theory as the correlations with the neutralino branching ratios.

This paper is organized as follows: We first define our models in Section 2. In Section 3 we present our on-shell scheme for the neutralino and chargino mass matrices in the considered models. Afterwards we present numerical results for the neutrino masses and mixings comparing the on-shell scheme and the $\overline{\text{DR}}$ scheme in Section 4. Next we discuss the decays $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ and the correlations to neutrino physics at tree level in Section 5. In Section 6 we first discuss the complete NLO corrections to the widths and discuss the correlations at one-loop level. Finally we conclude in Section 7. Appendix A gives the implementation of the considered models in the public program `CNNDecays`.

2. Model definition

In this section we introduce the models by presenting the superpotential and the soft-breaking terms. Moreover we will give the neutralino and chargino mass matrices at tree level and comment on the need of one-loop corrections for the neutrino masses.

2.1. Superpotential, soft terms and scalar potential

We will start with the superpotential of bilinear R -parity violation (BRpV) and the $\mu\nu\text{SSM}$ with one right-handed neutrino superfield $\widehat{\nu}^c$. Both model have in common the Yukawa part of the matter superfields coupled to the $SU(2)$ -doublet Higgs fields

$$\mathcal{W}_Y = \epsilon_{\alpha\beta} \left(Y_U^{ij} \widehat{Q}_i^\alpha \widehat{u}_j^c \widehat{H}_u^\beta + Y_D^{ij} \widehat{Q}_i^\beta \widehat{d}_j^c \widehat{H}_d^\alpha + Y_l^{ij} \widehat{L}_i^\beta \widehat{l}_j^c \widehat{H}_d^\alpha \right) \quad , \quad (1)$$

where $\epsilon_{\alpha\beta}$ is the complete antisymmetric $SU(2)$ tensor with $\epsilon_{12} = 1$ and i, j denote the three families. In case of BRpV the explicit μ and ϵ_i -terms are added and the total superpotential reads as

$$\mathcal{W}_{bR_p} = \mathcal{W}_Y + \epsilon_{\alpha\beta} \left[\epsilon_i \widehat{L}_i^\alpha \widehat{H}_u^\beta - \mu \widehat{H}_d^\alpha \widehat{H}_u^\beta \right] \quad , \quad (2)$$

whereas in the $\mu\nu\text{SSM}$ the Higgs doublets are coupled to the right-handed neutrino superfield $\widehat{\nu}^c$ and in addition a term containing the neutrino Yukawa coupling Y_ν is added

$$\mathcal{W}_{\mu\nu\text{SSM}} = \mathcal{W}_Y + \epsilon_{\alpha\beta} \left[Y_\nu^i \widehat{L}_i^\alpha \widehat{\nu}^c \widehat{H}_u^\beta - \lambda \widehat{\nu}^c \widehat{H}_d^\alpha \widehat{H}_u^\beta \right] + \frac{1}{3} \kappa \widehat{\nu}^c \widehat{\nu}^c \widehat{\nu}^c \quad . \quad (3)$$

The first term in Equation (2) and the last two terms in Equation (3) explicitly break lepton number if one assigns lepton number to $\widehat{\nu}^c$. If not the first term in Equation (3) explicitly breaks lepton number and one immediately sees that in both models one has the same number of R_p violating parameters. Note that for the phenomenology it does not matter if $\widehat{\nu}^c$ carries lepton number as it is broken explicitly by a least one interaction of this field. Both models have in common the soft SUSY breaking terms

$$\begin{aligned} V_{\text{soft}}^{\text{MSSM}-B\mu} = & m_Q^{ij2} \tilde{Q}_i^{\alpha*} \tilde{Q}_j^\alpha + m_U^{ij2} \tilde{u}_i^c \tilde{u}_j^{c*} + m_D^{ij2} \tilde{d}_i^c \tilde{d}_j^{c*} + m_L^{ij2} \tilde{L}_i^{\alpha*} \tilde{L}_j^\alpha + m_E^{ij2} \tilde{l}_i^c \tilde{l}_j^{c*} \\ & + m_{H_d}^2 H_d^{a*} H_d^a + m_{H_u}^2 H_u^{a*} H_u^a - \frac{1}{2} [M_1 \tilde{B}^0 \tilde{B}^0 + M_2 \tilde{W}^\gamma \tilde{W}^\gamma + M_3 \tilde{g}^{\gamma'} \tilde{g}^{\gamma'} + h.c.] \end{aligned}$$

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$$+ \epsilon_{\alpha\beta} [T_U^{ij} \tilde{Q}_i^\alpha \tilde{u}_j^{c*} H_u^\beta + T_D^{ij} \tilde{Q}_i^\beta \tilde{d}_j^{c*} H_d^\alpha + T_E^{ij} \tilde{L}_i^\beta \tilde{l}_j^{c*} H_d^\alpha + h.c.] \quad , \quad (4)$$

where $V_{soft}^{\text{MSSM}-B\mu}$ contains all the usual soft terms of the MSSM but the B_μ -term and a summation over $\gamma = 1, \dots, 3$ and $\gamma' = 1, \dots, 8$ has to be performed. In addition we add in case of BRpV

$$V_{soft}^{bR_p} = V_{soft}^{\text{MSSM}-B\mu} + \epsilon_{\alpha\beta} [B_i \epsilon_i \tilde{L}_i^\alpha H_u^\beta - B_\mu H_d^\alpha H_u^\beta + h.c.] \quad (5)$$

and in case of the $\mu\nu\text{SSM}$:

$$V_{soft}^{\mu\nu\text{SSM}} = V_{soft}^{\text{MSSM}-B\mu} + m_{\tilde{\nu}^c}^2 \tilde{\nu}^c \tilde{\nu}^{c*} + \epsilon_{\alpha\beta} [T_\nu^i \tilde{L}_i^\alpha \tilde{\nu}^c H_u^\beta - T_\lambda \tilde{\nu}^c H_d^\alpha H_u^\beta + h.c.] + [\frac{1}{3} T_\kappa \tilde{\nu}^c \tilde{\nu}^c \tilde{\nu}^c + h.c.] \quad (6)$$

In these expressions the notation for the soft trilinear couplings introduced in [52] is used. We will not present the scalar potential and the corresponding tadpole equations in this paper, but refer to [30] for BRpV and to [47] for the $\mu\nu\text{SSM}$. Note the different definition of κ in comparison to [47]. The determination of the vacuum structure through the scalar potential induces the following vacuum expectation values (VEVs):

$$\langle H_d^0 \rangle = \frac{1}{\sqrt{2}} v_d, \quad \langle H_u^0 \rangle = \frac{1}{\sqrt{2}} v_u, \quad \langle \tilde{\nu}_i \rangle = \frac{1}{\sqrt{2}} v_i, \quad \langle \tilde{\nu}^c \rangle = \frac{1}{\sqrt{2}} v_c \quad (7)$$

Again $\tilde{\nu}^c$ is only present in the $\mu\nu\text{SSM}$, where its vacuum expectation value generates effective μ - and ϵ_i -terms

$$\mu = \frac{1}{\sqrt{2}} \lambda v_c, \quad \epsilon_i = \frac{1}{\sqrt{2}} Y_\nu^i v_c \quad . \quad (8)$$

In this way $\hat{\nu}^c$ plays the role of the singlet superfield \hat{S} in the NMSSM, implying that μ , but also ϵ_i is naturally of the order of the electroweak scale [43]. Moreover there are no additional terms with dimension of mass present in the superpotential. Beside the parameter ϵ_i we define the so-called alignment parameters

$$\Lambda_i = \mu v_i + v_d \epsilon_i \quad , \quad (9)$$

which determine the neutrino mass matrix at tree level [47] and which give the relevant information on the tree level mass matrix in a basis invariant way [53]. ϵ_i and Λ_i can then be used to fit the neutrino data. Concerning the scalar potential we add, that in case of BRpV we determine the soft-breaking parameters $m_{H_d}^2, m_{H_u}^2$ and B_i and in case of the $\mu\nu\text{SSM}$ $m_{H_d}^2, m_{H_u}^2, m_{\tilde{\nu}^c}^2$ and T_ν^i from the tadpole equations.

2.2. Masses of neutralinos and charginos

Using Weyl spinors defined by

$$(\psi^-)^T = (\tilde{W}^-, \tilde{H}_d^-, e, \mu, \tau), \quad (\psi^+)^T = (\tilde{W}^+, \tilde{H}_u^+, e^c, \mu^c, \tau^c) \quad (10)$$

the Lagrangian density containing the chargino masses can be written in the form

$$\mathcal{L} \supset -\frac{1}{2} ((\psi^-)^T \mathcal{M}_c \psi^+ + (\psi^+)^T (\mathcal{M}_c)^T \psi^-) + h.c. \quad , \quad (11)$$

where the mass matrix of the charged fermions is given by

$$\mathcal{M}_c = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g v_u & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} g v_d & \mu & -\frac{1}{\sqrt{2}} Y_l^{i1} v_i & -\frac{1}{\sqrt{2}} Y_l^{i2} v_i & -\frac{1}{\sqrt{2}} Y_l^{i3} v_i \\ \frac{1}{\sqrt{2}} g v_1 & -\epsilon_1 & \frac{1}{\sqrt{2}} Y_l^{11} v_d & \frac{1}{\sqrt{2}} Y_l^{12} v_d & \frac{1}{\sqrt{2}} Y_l^{13} v_d \\ \frac{1}{\sqrt{2}} g v_2 & -\epsilon_2 & \frac{1}{\sqrt{2}} Y_l^{21} v_d & \frac{1}{\sqrt{2}} Y_l^{22} v_d & \frac{1}{\sqrt{2}} Y_l^{23} v_d \\ \frac{1}{\sqrt{2}} g v_3 & -\epsilon_3 & \frac{1}{\sqrt{2}} Y_l^{31} v_d & \frac{1}{\sqrt{2}} Y_l^{32} v_d & \frac{1}{\sqrt{2}} Y_l^{33} v_d \end{pmatrix} . \quad (12)$$

The mass eigenstates are obtained by two unitary matrices U and V

$$F_i^+ = V_{it} \psi_t^+ \quad \text{and} \quad F_i^- = U_{it} \psi_t^- \quad , \quad (13)$$

so that the mass eigenvalues can be calculated via

$$\mathcal{M}_{c,dia.} = \text{Diag} \left(m_{\tilde{\chi}_1^+}, \dots, m_{\tilde{\chi}_5^+} \right) = U^* \mathcal{M}_c V^{-1} \quad (14)$$

$$m_{\tilde{\chi}_i^+} \leq m_{\tilde{\chi}_j^+} \quad \text{for} \quad i < j$$

and the Dirac spinor $\tilde{\chi}_i^+$ can be deduced from

$$\tilde{\chi}_i^+ = \begin{pmatrix} F_i^+ \\ F_i^- \end{pmatrix} . \quad (15)$$

For the neutralinos we obtain in the basis ²

$$(\psi^0)^T = \left(\tilde{B}^0, \tilde{W}_3^0, \tilde{H}_d^0, \tilde{H}_u^0, \nu^c, \nu_1, \nu_2, \nu_3 \right) \quad (16)$$

the mass matrices of the neutral fermions, which have the generic structure

$$\mathcal{M}_n = \begin{pmatrix} M_H & \hat{m} \\ \hat{m}^T & 0 \end{pmatrix} \quad (17)$$

and enter the Lagrangian density $\mathcal{L} \supset -\frac{1}{2} ((\psi^0)^T \mathcal{M}_n \psi^0) + h.c.$. The submatrix M_H contains the 4(5) heavy states, which are the four usual neutralinos of the MSSM and in case of the $\mu\nu$ SSM in addition the right-handed neutrino. The submatrix \hat{m} mixes the heavy states with the left-handed neutrinos and contains the R -parity breaking parameters. In detail the elements are given by

$$M_H = \begin{pmatrix} M_1 & 0 & -\frac{1}{2} g' v_d & \frac{1}{2} g' v_u & 0 \\ 0 & M_2 & \frac{1}{2} g v_d & -\frac{1}{2} g v_u & 0 \\ -\frac{1}{2} g' v_d & \frac{1}{2} g v_d & 0 & -\mu & -\frac{1}{\sqrt{2}} \lambda v_u \\ \frac{1}{2} g' v_u & -\frac{1}{2} g v_u & -\mu & 0 & -\frac{1}{\sqrt{2}} \lambda v_d \\ 0 & 0 & -\frac{1}{\sqrt{2}} \lambda v_u & -\frac{1}{\sqrt{2}} \lambda v_d & \sqrt{2} \kappa v_c \end{pmatrix} \quad (18)$$

² $\hat{\nu}^c$ is only present in the $\mu\nu$ SSM.

$$\hat{m}^T = \begin{pmatrix} -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & \epsilon_1 & \epsilon_1 \frac{v_u}{v_c} \\ -\frac{1}{2}g'v_2 & \frac{1}{2}gv_2 & 0 & \epsilon_2 & \epsilon_2 \frac{v_u}{v_c} \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 & \epsilon_3 \frac{v_u}{v_c} \end{pmatrix}, \quad (19)$$

where in the $\mu\nu$ SSM μ and ϵ_i can be taken from Equation (8). In case of BRpV one has to omit the last column and the last row in the submatrix M_H and the last column in the matrix \hat{m} . The mass eigenstates F_i^0 are obtained via

$$F_i^0 = \mathcal{N}_{is}\psi_s^0 \quad (20)$$

from the gauge eigenstates ψ_s^0 , where the unitary matrix \mathcal{N} diagonalizes the mass matrix \mathcal{M}_n as

$$\mathcal{M}_{n,dia.} = \text{Diag}\left(m_{\tilde{\chi}_1^0}, \dots, m_{\tilde{\chi}_j^0}\right) = \mathcal{N}^* \mathcal{M}_n \mathcal{N}^\dagger, \quad (21)$$

where $j = 7(8)$ in case of BRpV ($\mu\nu$ SSM). The masses are ordered as $m_{\tilde{\chi}_i^0} \leq m_{\tilde{\chi}_j^0}$ for $i < j$ and we have chosen \mathcal{N} such that all mass eigenvalues are positive. This implies that in general the matrix \mathcal{N} is complex. Similar to [45] we can do useful checks using negative mass eigenvalues in combination with a real mixing matrix \mathcal{N} . The 4-component spinor can be obtained via

$$\tilde{\chi}_i^0 = \begin{pmatrix} F_i^0 \\ F_i^0 \end{pmatrix}. \quad (22)$$

All the other masses of the scalar sectors can be taken from [30] for BRpV and from [47] for the $\mu\nu$ SSM. The various couplings, which are needed for the one-loop calculations, can be found in the folder couplings within **CNNDecays** [51].

2.3. Neutrino masses

Due to the smallness of the R -parity breaking parameters an effective neutrino mass matrix in a seesaw approximation can be found

$$m_{\nu\nu}^{\text{eff}} = -\hat{m}^T M_H^{-1} \hat{m} = -\xi \hat{m}, \quad (23)$$

where the matrix ξ contains the small expansion parameters and can be taken from [30, 47]. The basic calculation yields

$$(m_{\nu\nu}^{\text{eff}})_{ij} = a\Lambda_i\Lambda_j \quad \text{with } a \text{ being} \quad (24)$$

$$a^{bR_p} = \frac{m_\gamma}{4\text{Det}_0^{bR_p}}, \quad a^{\mu\nu\text{SSM}} = \frac{m_\gamma(\lambda^2 v_d v_u + \mu m_R)}{4\mu\text{Det}_0^{\mu\nu\text{SSM}}}, \quad (25)$$

where we have used the abbreviations in Equations (8) and (9) and

$$m_\gamma = g^2 M_1 + g'^2 M_2, \quad m_R = \sqrt{2}\kappa v_c, \quad (26)$$

where the singlet mass parameter m_R is only valid in the $\mu\nu$ SSM. The determinants of M_H are given by

$$\text{Det}_0^{bR_p} = \frac{1}{2}m_\gamma\mu v_d v_u - M_1 M_2 \mu^2 \quad (27)$$

$$\text{Det}_0^{\mu\nu\text{SSM}} = \frac{1}{8}m_\gamma(\lambda^2 v^4 + 4m_R\mu v_d v_u) - M_1 M_2 \mu(v_d v_u \lambda^2 + m_R \mu) \quad . \quad (28)$$

The projective form of this mass matrix implies that only one neutrino acquires a tree level mass, while the other two remain massless. We denote the matrix, which diagonalizes $m_{\nu\nu}^{\text{eff}}$, in the following \mathcal{N}_ν . The correct magnitude of $\vec{\Lambda}$ can be easily estimated from the approximated formula for the neutrino mass m_{ν_3} at tree level

$$m_{\nu_3} = a|\vec{\Lambda}| \quad . \quad (29)$$

Since one neutrino mass is not sufficient to explain the oscillation data, one-loop corrections in BRpV [30, 31, 54] and in the $\mu\nu\text{SSM}$ [47, 48, 55] have to be added to match the current bounds. We will therefore consider the complete mass matrices at the one-loop level using an on-shell scheme for calculating the masses. In addition we will show, that this also requires the complete NLO corrections to the decays $\Gamma(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp)$ to obtain the correct correlations between neutrino mixing angles and ratios of branching ratios.

3. One-loop on-shell masses of neutralinos and charginos

As we have shown pure tree level neutralino masses including the neutrinos cannot explain the full neutrino bounds in BRpV and the $\mu\nu\text{SSM}$ with one right-handed neutrino $\hat{\nu}^c$ only. We will therefore go to the next order in perturbation theory for the masses, which was illustrated using a $\overline{\text{DR}}$ scheme in BRpV [30, 31, 54] and in the $\mu\nu\text{SSM}$ [47, 48, 55] in previous articles. However, we will later focus on the decay width $\Gamma(\tilde{\chi}_1^0 \rightarrow l_i^\pm W^\mp)$ at one-loop level to investigate the relations between branching ratios and neutrino mixing angles on higher loop order and choose a more convenient on-shell scheme for our calculation. Before discussing this topic, we will first discuss the masses for neutralinos and charginos in an on-shell scheme as in [45], which allows to have finite corrections to the neutrino masses $m_{\nu_i}^{1L}$ but also to the mixing matrix \mathcal{N}^{1L} at one-loop level, which is crucial to explain the neutrino oscillation data. The approach is similar to [56, 57], whereas for the MSSM another procedure has been worked out in [58, 59].

3.1. Common basic principles

A special feature of the chargino/neutralino sector is, that the number of parameters is lower than the number of imposed on-shell conditions. This implies finite corrections to the tree level masses of neutralinos and charginos. We closely follow [45, 56] and start with the one-loop contributions to neutralino masses $\delta\mathcal{M}_n^\oplus$ and chargino masses $\delta\mathcal{M}_c^\oplus$

$$(\delta\mathcal{M}_n^\oplus)_{ij} = \delta(\mathcal{N}^T \mathcal{M}_{n,dia}^\oplus \mathcal{N})_{ij} \quad (30)$$

$$= \sum_{n,l} \left[\delta\mathcal{N}_{ni} (\mathcal{M}_{n,dia}^\oplus)_{nl} \mathcal{N}_{lj} + \mathcal{N}_{ni} (\delta\mathcal{M}_{n,dia}^\oplus)_{nl} \mathcal{N}_{lj} + \mathcal{N}_{ni} (\mathcal{M}_{n,dia}^\oplus)_{nl} \delta\mathcal{N}_{lj} \right]$$

$$(\delta\mathcal{M}_c^\oplus)_{ij} = \delta(U^T \mathcal{M}_{c,dia}^\oplus V)_{ij} \quad (31)$$

$$= \sum_{n,l} \left[\delta U_{ni} (\mathcal{M}_{c,dia}^\oplus)_{nl} V_{lj} + U_{ni} (\delta\mathcal{M}_{c,dia}^\oplus)_{nl} V_{lj} + U_{ni} (\mathcal{M}_{c,dia}^\oplus)_{nl} \delta V_{lj} \right]$$

with the diagonalized mass matrices $\mathcal{M}_{dia.}^{\otimes}$ and their counterterms $\delta\mathcal{M}_{dia.}^{\otimes}$:

$$(\mathcal{M}_{n,dia.}^{\otimes})_{nl} = \delta_{nl}m_{\tilde{\chi}_l^0}, \quad (\delta\mathcal{M}_{n,dia.}^{\otimes})_{nl} = \delta_{nl}\delta m_{\tilde{\chi}_l^0} \quad (32)$$

$$(\mathcal{M}_{c,dia.}^{\otimes})_{nl} = \delta_{nl}m_{\tilde{\chi}_l^{\pm}}, \quad (\delta\mathcal{M}_{c,dia.}^{\otimes})_{nl} = \delta_{nl}\delta m_{\tilde{\chi}_l^{\pm}} \quad (33)$$

The bare mass matrices of the neutralinos and charginos can now be expressed as the corrected on-shell mass matrix with the corrections in Equations (30) and (31) or via the tree level mass matrix expressed in physical parameters together with the renormalization constants of those:

$$\mathcal{M}_{n,c}^0 = \mathcal{M}_{n,c}^{\otimes} + \delta\mathcal{M}_{n,c}^{\otimes} = \mathcal{M}_{n,c} + \delta\mathcal{M}_{n,c} \quad (34)$$

Therefore the relations between tree level and one-loop mass matrices take the form:

$$\mathcal{M}_{n,c}^{\otimes} = \mathcal{M}_{n,c} + \delta\mathcal{M}_{n,c} - \delta\mathcal{M}_{n,c}^{\otimes} =: \mathcal{M}_{n,c} + \Delta\mathcal{M}_{n,c} \quad (35)$$

In the following we will define the model-dependent physical parameters, write down the renormalization of the mass matrices $\delta\mathcal{M}_{n,c}$ and identify the renormalization constants of the physical parameters, which should be fixed in the neutralino or chargino sector. Some physical parameters, namely $\delta m_W, \delta m_Z$ and thus $\delta \cos \theta_W$ are fixed in the gauge boson sector and $\delta \tan \beta$ in the Higgs sector. In particular for $\delta \tan \beta$ we take the $\overline{\text{DR}}$ renormalization [60], so that UV divergences in the masses and the considered process cancel

$$\frac{\delta \tan \beta}{\tan \beta} = \frac{1}{32\pi^2} \Delta \left(3\text{Tr}(Y_d Y_d^\dagger) - 3\text{Tr}(Y_u Y_u^\dagger) + \text{Tr}(Y_l Y_l^\dagger) - \text{Tr}(Y_\nu Y_\nu^\dagger) \right) \quad , \quad (36)$$

where $\Delta = \frac{1}{\epsilon} - \gamma + \ln(4\pi)$ parametrizes the UV divergence as in [45] and Y_ν is only present in the $\mu\nu\text{SSM}$. Note that this choice maintains also the gauge invariance of masses and the considered decay widths. We will also need the renormalization constants for the sine and cosine of different angles, which are given by:

$$\frac{\delta \cos \beta}{\cos \beta} = -\sin^2 \beta \frac{\delta \tan \beta}{\tan \beta}, \quad \frac{\delta \sin \beta}{\sin \beta} = \cos^2 \beta \frac{\delta \tan \beta}{\tan \beta} \quad (37)$$

3.2. Bilinear R -parity breaking

The on-shell renormalization of neutralino and chargino masses in the MSSM and NMSSM was extensively discussed in [45]. In case of BRpV the renormalization of the physical parameters is more challenging. The masses of the Z and W bosons are defined in the following form

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2) \left(v_d^2 + v_u^2 + \sum_i v_i^2 \right), \quad m_W^2 = \frac{1}{4}g^2 \left(v_d^2 + v_u^2 + \sum_i v_i^2 \right) \quad , \quad (38)$$

where not only the vacuum expectation values of the Higgs enter, but also the ones of the left-handed sneutrinos. As in the (N)MSSM the cos of the given Weinberg angle $\cos \theta_W$ can be derived from $m_W = \cos \theta_W m_Z$. Beside the already known $\tan \beta$, we define

$$\tan \beta_i = \frac{v_i}{v_d} \quad \text{and} \quad \epsilon_i \quad (39)$$

as additional parameters, which are present at tree level, such that the tree level mass matrix of the neutralinos is given by

$$\mathcal{M}_n = \begin{pmatrix} M_H & \hat{m} \\ \hat{m}^T & 0 \end{pmatrix} \quad (40)$$

$$M_H = \begin{pmatrix} M_1 & 0 & -m_Z \sin \theta_W \cos \beta \Theta & m_Z \sin \theta_W \sin \beta \Theta \\ & M_2 & m_Z \cos \theta_W \cos \beta \Theta & -m_Z \cos \theta_W \sin \beta \Theta \\ & & 0 & -\mu \\ \text{sym.} & & & 0 \end{pmatrix} \quad (41)$$

$$(\hat{m}^T)_i = (-m_Z \sin \theta_W \cos \beta \tan \beta_i \Theta \quad m_Z \cos \theta_W \cos \beta \tan \beta_i \Theta \quad 0 \quad \epsilon_i) \quad , \quad (42)$$

where Θ is a function of $\tan \beta$ and $\tan \beta_i$, namely:

$$\Theta = \Theta(\beta, \beta_i) = \sqrt{\frac{1}{1 + \cos^2 \beta \sum_i \tan^2 \beta_i}} \xrightarrow{v_i \rightarrow 0} 1 \quad (43)$$

In this way we maintain the possibility to fix the renormalization constants of m_Z and $\cos \theta_W$ in the gauge boson sector, whereas the corrections from R -parity breaking are parameterized by $\tan \beta_i$, ϵ_i and the parameter Θ , the latter depends on β and β_i . Defining in addition the lepton masses $m_{ij} = \frac{1}{\sqrt{2}} Y_l^{ij} v_d$ the chargino mass matrix is given by:

$$\mathcal{M}_c = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta \Theta & 0 & 0 & 0 \\ \sqrt{2} m_W \cos \beta \Theta & \mu & -\tan \beta_i m_{i1} & -\tan \beta_i m_{i2} & -\tan \beta_i m_{i3} \\ \sqrt{2} m_W \cos \beta \tan \beta_1 \Theta & -\epsilon_1 & m_{11} & m_{12} & m_{13} \\ \sqrt{2} m_W \cos \beta \tan \beta_2 \Theta & -\epsilon_2 & m_{21} & m_{22} & m_{23} \\ \sqrt{2} m_W \cos \beta \tan \beta_3 \Theta & -\epsilon_3 & m_{31} & m_{32} & m_{33} \end{pmatrix} \quad (44)$$

We discuss the general case of non-diagonal lepton masses m_{ij} , since the counterterms δm_{ij} are needed to cancel non-diagonal contributions in the lepton mass matrix at one-loop level. However, the lepton Yukawa couplings Y_l and thus m_{ij} can be chosen diagonal at tree level. Using now the relations $\cos \beta^0 = \cos \beta + \delta \cos \beta$ and $\tan \beta_i^0 = \tan \beta_i + \delta \tan \beta_i$ one can do a simple expansion in first order of $\delta \cos \beta$ and $\delta \tan \beta_i$:

$$\begin{aligned} \Theta_0 &= \sqrt{\frac{1}{1 + \cos^2 \beta^0 \sum_i \tan^2 \beta_i^0}} \approx \sqrt{\frac{1}{1 + \cos^2 \beta \sum_i \tan^2 \beta_i}} \\ &\quad - \left(\frac{1}{1 + \cos^2 \beta \sum_j \tan^2 \beta_j} \right)^{\frac{3}{2}} \sum_i (\cos^2 \beta \tan \beta_i \delta \tan \beta_i + \cos \beta \delta \cos \beta \tan^2 \beta_i) \\ &= \Theta + \delta \Theta \end{aligned} \quad (45)$$

Therefore the counterterm of Θ can be expressed as a function of $\delta \tan \beta_i$ and $\delta \cos \beta$:

$$\delta \Theta = -\cos \beta \delta \cos \beta \Theta^3 \sum_i \tan^2 \beta_i - \cos^2 \beta \Theta^3 \sum_i \tan \beta_i \delta \tan \beta_i \quad (46)$$

$$= - \sum_i \cos^2 \beta \tan^2 \beta_i \Theta^3 \left(\frac{\delta \cos \beta}{\cos \beta} + \frac{\delta \tan \beta_i}{\tan \beta_i} \right) \xrightarrow{v_i \rightarrow 0} 0$$

The variation of all non-zero entries of the tree level mass matrix yields:

$$\delta \mathcal{M}_n^{11} = \delta M_1 = \frac{\delta M_1}{M_1} \mathcal{M}_n^{11} \quad (47)$$

$$\delta \mathcal{M}_n^{13} = -\delta(m_Z \sin \theta_W \cos \beta \Theta) = \left(\frac{\delta m_Z}{m_Z} + \frac{\delta \sin \theta_W}{\sin \theta_W} + \frac{\delta \cos \beta}{\cos \beta} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_n^{13} \quad (48)$$

$$\delta \mathcal{M}_n^{14} = \delta(m_Z \sin \theta_W \sin \beta \Theta) = \left(\frac{\delta m_Z}{m_Z} + \frac{\delta \sin \theta_W}{\sin \theta_W} + \frac{\delta \sin \beta}{\sin \beta} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_n^{14} \quad (49)$$

$$\begin{aligned} \delta \mathcal{M}_n^{1,4+i} &= -\delta(m_Z \sin \theta_W \cos \beta \tan \beta_i \Theta) \\ &= \left(\frac{\delta m_Z}{m_Z} + \frac{\delta \sin \theta_W}{\sin \theta_W} + \frac{\delta \cos \beta}{\cos \beta} + \frac{\delta \tan \beta_i}{\tan \beta_i} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_n^{1,4+i} \end{aligned} \quad (50)$$

$$\delta \mathcal{M}_n^{22} = \delta M_2 = \frac{\delta M_2}{M_2} \mathcal{M}_n^{22} \quad (51)$$

$$\delta \mathcal{M}_n^{23} = \delta(m_Z \cos \theta_W \cos \beta \Theta) = \left(\frac{\delta m_Z}{m_Z} + \frac{\delta \cos \theta_W}{\cos \theta_W} + \frac{\delta \cos \beta}{\cos \beta} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_n^{23} \quad (52)$$

$$\delta \mathcal{M}_n^{24} = -\delta(m_Z \cos \theta_W \sin \beta \Theta) = \left(\frac{\delta m_Z}{m_Z} + \frac{\delta \cos \theta_W}{\cos \theta_W} + \frac{\delta \sin \beta}{\sin \beta} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_n^{24} \quad (53)$$

$$\begin{aligned} \delta \mathcal{M}_n^{2,4+i} &= \delta(m_Z \cos \theta_W \cos \beta \tan \beta_i \Theta) \\ &= \left(\frac{\delta m_Z}{m_Z} + \frac{\delta \cos \theta_W}{\cos \theta_W} + \frac{\delta \cos \beta}{\cos \beta} + \frac{\delta \tan \beta_i}{\tan \beta_i} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_n^{2,4+i} \end{aligned} \quad (54)$$

$$\delta \mathcal{M}_n^{34} = -\delta \mu = \frac{\delta \mu}{\mu} \mathcal{M}_n^{34} \quad (55)$$

$$\delta \mathcal{M}_n^{4,4+i} = \delta \epsilon_i = \frac{\delta \epsilon_i}{\epsilon_i} \mathcal{M}_n^{4,4+i} \quad (56)$$

whereas all the other variations $\delta \mathcal{M}_n^{12} = \delta \mathcal{M}_n^{33} = \delta \mathcal{M}_n^{3,4+i} = \delta \mathcal{M}_n^{44} = 0$ necessarily vanish. The corrections in the chargino mass matrix read:

$$\delta \mathcal{M}_c^{11} = \delta M_2 = \frac{\delta M_2}{M_2} \mathcal{M}_c^{22} \quad (57)$$

$$\delta \mathcal{M}_c^{12} = \sqrt{2} \delta(m_W \sin \beta \Theta) = \left(\frac{\delta m_W}{m_W} + \frac{\delta \sin \beta}{\sin \beta} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_c^{12} \quad (58)$$

$$\delta \mathcal{M}_c^{21} = \sqrt{2} \delta(m_W \cos \beta \Theta) = \left(\frac{\delta m_W}{m_W} + \frac{\delta \cos \beta}{\cos \beta} + \frac{\delta \Theta}{\Theta} \right) \mathcal{M}_c^{21} \quad (59)$$

$$\delta \mathcal{M}_c^{22} = \delta \mu = \frac{\delta \mu}{\mu} \mathcal{M}_c^{22} \quad (60)$$

$$\delta \mathcal{M}_c^{2,2+i} = \delta(-\tan \beta_k m_{ki}) = \sum_k \left(\frac{\delta \tan \beta_k}{\tan \beta_k} - \frac{\delta m_{ki}}{m_{ki}} \right) \mathcal{M}_c^{2,2+i} \quad (61)$$

$$\delta\mathcal{M}_c^{2+i,1} = \sqrt{2}\delta(m_W \cos\beta \tan\beta_i \Theta) \quad (62)$$

$$= \left(\frac{\delta m_W}{m_W} + \frac{\delta \cos\beta}{\cos\beta} + \frac{\delta \tan\beta_i}{\tan\beta_i} + \frac{\delta\Theta}{\Theta} \right) \mathcal{M}_c^{2+i,1}$$

$$\delta\mathcal{M}_c^{2+i,2} = -\delta\epsilon_i = \frac{\delta\epsilon_i}{\epsilon_i} \mathcal{M}_c^{2+i,2} \quad (63)$$

$$\delta\mathcal{M}_c^{2+i,2+j} = \delta m_{ij} = \frac{\delta m_{ij}}{m_{ij}} \mathcal{M}_c^{2+i,2+j} \quad (64)$$

and $\delta\mathcal{M}_c^{1,2+i} = 0$ vanishes. Similar to the (N)MSSM we fix δM_1 in the neutralino and $\delta M_2, \delta\mu$ in the chargino sector. However, we still have to find appropriate renormalization conditions for $\delta \tan\beta_i, \delta\epsilon_i$ and δm_{ij} . Whereas δm_{ij} should be fixed in the lepton sector, so that one-loop contributions to leptonic two-point functions vanish, we have several possibilities for $\delta \tan\beta_i$ and $\delta\epsilon_i$. Whereas $\delta \tan\beta_i$ could for example be fixed in the Higgs sector together with $\tan\beta$, one could fix $\delta\epsilon_i$ with respect to an R -parity violating decay. We will focus on stable lepton masses. Thus, we calculate $\delta \tan\beta_i$ and $\delta\epsilon_i$ from the one-loop contributions in the chargino sector, which mix the well-known MSSM charginos with the leptons. In total we therefore impose the following conditions for the MSSM parameters

$$\Delta\mathcal{M}_n^{11} = \Delta\mathcal{M}_c^{11} = \Delta\mathcal{M}_c^{22} \stackrel{!}{=} 0 \quad , \quad (65)$$

which results in

$$\delta M_1 = \delta\mathcal{M}_n^{\otimes 11}, \quad \delta M_2 = \delta\mathcal{M}_c^{\otimes 11}, \quad \delta\mu = -\delta\mathcal{M}_c^{\otimes 22} \quad . \quad (66)$$

In a second step the conditions $\Delta\mathcal{M}_c^{2+i,2+j} \stackrel{!}{=} 0$ for the renormalization constants of the lepton masses δm_{ij} are imposed, resulting in

$$\delta m_{ii} = \delta\mathcal{M}_c^{\otimes 2+i,2+j} \quad . \quad (67)$$

In a last step the R -parity violating sector with $\delta \tan\beta_i$ and $\delta\epsilon_i$ is considered, which can be fixed by imposing $\Delta\mathcal{M}_c^{2+i,2} = \Delta\mathcal{M}_c^{2,2+i} \stackrel{!}{=} 0$

$$\delta \tan\beta_i = \frac{1}{\det(m)} \frac{1}{2} \sum_{j,k,l,r,s} \epsilon_{ijk} \epsilon_{lrs} \Upsilon_l m_{jr} m_{ks} \quad (68)$$

$$\text{with } \Upsilon_i = - \sum_k \tan\beta_k \delta m_{ki} - \delta\mathcal{M}_n^{\otimes 2,2+i} \quad (69)$$

$$\delta\epsilon_i = \delta\mathcal{M}_c^{\otimes 2+i,2} \quad , \quad (70)$$

where ϵ_{ijk} is the Levi-Civita symbol and $\det(m)$ is the determinant of the lepton masses. In case of vanishing non-diagonal lepton masses at tree level $m_{ij} = 0$ for $i \neq j$ this simplifies to $\delta \tan\beta_i = \frac{1}{m_{ii}} \Upsilon_i$. Another possibility is to fix $\delta \tan\beta_i$ from $\Delta\mathcal{M}_c^{2+i,1}$. However, this induces a dependence on the renormalization constants of the gauge sector, whereas in the described case the neutrino and lepton sector are “decoupled” from those.

For all the remaining entries of the neutralino and chargino mass matrices shifts $\Delta\mathcal{M}_{n,c}$ have to be taken into account. Due to the non-vanishing entries $\Delta\mathcal{M}_c^{2+i,1}$ also the lepton masses differ between tree and one-loop level. However, we will show that this difference is tiny, for reasonable neutrino masses far below the experimental uncertainties. Note that the Yukawa couplings of the leptons at tree level of course have to be adopted, so that the tree level lepton masses fit the experimental known values.

3.3. $\mu\nu SSM$

Combining the already known procedures from the NMSSM and BRpV we define

$$\tan\beta_i = \frac{v_i}{v_d}, \quad \tan\beta_c = \frac{v_c}{v_u} \quad (71)$$

$$\mu = \frac{1}{\sqrt{2}}\lambda v_c, \quad m_R = \frac{1}{\sqrt{2}}\kappa v_c, \quad \epsilon_i = \frac{1}{\sqrt{2}}Y_\nu^i v_c \quad . \quad (72)$$

In this notation the tree level mass matrix of the neutralinos is given by

$$\mathcal{M}_n = \begin{pmatrix} M_H & \hat{m} \\ \hat{m}^T & 0 \end{pmatrix} \quad (73)$$

$$M_H = \begin{pmatrix} M_1 & 0 & -m_Z \sin\theta_W \cos\beta\Theta & m_Z \sin\theta_W \sin\beta\Theta & 0 \\ & M_2 & m_Z \cos\theta_W \cos\beta\Theta & -m_Z \cos\theta_W \sin\beta\Theta & 0 \\ & & 0 & -\mu & -\frac{\mu}{\tan\beta_c} \\ \text{sym.} & & & 0 & \frac{\tan\beta_i \epsilon_i - \mu}{\tan\beta \tan\beta_c} \\ & & & & m_R \end{pmatrix} \quad (74)$$

$$(\hat{m}^T)_i = \begin{pmatrix} -m_Z \sin\theta_W \cos\beta \tan\beta_i \Theta & m_Z \cos\theta_W \cos\beta \tan\beta_i \Theta & 0 & \epsilon_i & \frac{\epsilon_i}{\tan\beta_c} \end{pmatrix} \quad , \quad (75)$$

where Θ is defined in Equation (43) and the chargino mass matrix has the same form as in BRpV, see Equation (44). Beside the variations already present in the bilinear model as shown in Equations (47)-(56), where the indices $4+i$ have to be shifted to $5+i$, $\delta\mathcal{M}_n$ has the following additional entries:

$$\delta\mathcal{M}_n^{35} = \delta\left(-\frac{\mu}{\tan\beta_c}\right) = \left(\frac{\delta\mu}{\mu} - \frac{\delta\tan\beta_c}{\tan\beta_c}\right) \mathcal{M}_n^{35} \quad (76)$$

$$\delta\mathcal{M}_n^{45} = \delta\left(\frac{\tan\beta_i \epsilon_i - \mu}{\tan\beta \tan\beta_c}\right) = \left(-\frac{\delta\tan\beta}{\tan\beta} - \frac{\delta\tan\beta_c}{\tan\beta_c} + \frac{\delta\tan\beta_i \epsilon_i + \tan\beta_i \delta\epsilon_i - \delta\mu}{\tan\beta_i \epsilon_i - \mu}\right) \mathcal{M}_n^{45} \quad (77)$$

$$\delta\mathcal{M}_n^{55} = \frac{\delta m_R}{m_R} \mathcal{M}_n^{55} \quad (78)$$

$$\delta\mathcal{M}_n^{5,5+i} = \delta\left(\frac{\epsilon_i}{\tan\beta_c}\right) = \left(\frac{\delta\epsilon_i}{\epsilon_i} - \frac{\delta\tan\beta_c}{\tan\beta_c}\right) \mathcal{M}_n^{5,5+i} \quad (79)$$

whereas all the other variations $\delta\mathcal{M}_n^{12} = \delta\mathcal{M}_n^{15} = \delta\mathcal{M}_n^{25} = \delta\mathcal{M}_n^{33} = \delta\mathcal{M}_n^{3,5+i} = \delta\mathcal{M}_n^{44} = 0$ necessarily vanish. The corrections in the chargino mass matrix are the same as in the

bilinear case, presented in Equations (57)-(64). Similar to the MSSM, we fix δM_1 in the neutralino and $\delta M_2, \delta\mu$ in the chargino sector. Similar to the NMSSM, we fix $\delta \tan \beta_c$ and δm_R in the neutralino sector and similar to the BRpV case $\delta \tan \beta_i$ and $\delta \epsilon_i$ are determined in the chargino sector. However, we will summarize these results once again for the $\mu\nu$ SSM. We start with the conditions for the non- R -parity breaking variables

$$\Delta\mathcal{M}_c^{11} = \Delta\mathcal{M}_c^{22} = \Delta\mathcal{M}_n^{11} = \Delta\mathcal{M}_n^{35} = \Delta\mathcal{M}_n^{55} \stackrel{!}{=} 0 \quad , \quad (80)$$

which results in:

$$\delta M_1 = \delta\mathcal{M}_n^{\otimes 11}, \quad \delta M_2 = \delta\mathcal{M}_c^{\otimes 11}, \quad \delta\mu = \delta\mathcal{M}_c^{\otimes 22} \quad (81)$$

$$\delta \tan \beta_c = \frac{\tan^2 \beta_c}{\mu} \left(\delta\mathcal{M}_n^{\otimes 35} - \frac{1}{\tan \beta_c} \delta\mathcal{M}_n^{\otimes 34} \right), \quad \delta m_R = \delta\mathcal{M}_n^{\otimes 55} \quad (82)$$

In a second step the conditions $\Delta\mathcal{M}_c^{2+i,2} = \Delta\mathcal{M}_c^{2,2+i} = \Delta\mathcal{M}_c^{2+i,2+j} \stackrel{!}{=} 0$ for the renormalization constants of the lepton masses δm_{ij} , $\delta \tan \beta_i$ and $\delta \epsilon_i$ are imposed, resulting in:

$$\delta m_{ij} = \delta\mathcal{M}_c^{\otimes 2+i,2+j}, \quad \delta \epsilon_i = \delta\mathcal{M}_c^{\otimes 2+i,2} \quad (83)$$

$$\delta \tan \beta_i = \frac{1}{\det(m)} \frac{1}{2} \sum_{j,k,l,r,s} \epsilon_{ijk} \epsilon_{lrs} \Upsilon_l m_{jr} m_{ks} \quad (84)$$

$$\text{with } \Upsilon_i = - \sum_k \tan \beta_k \delta m_{ki} - \delta\mathcal{M}_n^{\otimes 2,2+i} \quad (85)$$

3.4. Definition of one-loop on-shell masses

With the procedure introduced in the last sections we can calculate one-loop on-shell neutralino and chargino masses for BRpV and the $\mu\nu$ SSM, namely by combining the full one-loop corrections $\delta\mathcal{M}_{n,c}^{\otimes}$ with the counterterms $\delta\mathcal{M}_{n,c}$ obtained according to Equation (35). This results in the one-loop mass matrix $\mathcal{M}_{n,c}^{\otimes}$, whose diagonalizations lead to one-loop neutralino $m_{\tilde{\chi}_i^0}^{1L}$ and chargino masses $m_{\tilde{\chi}_i^\pm}^{1L}$ and mixing matrices at the one-loop level $\mathcal{N}^{1L}, U^{1L}, V^{1L}$. Note that these masses are UV and IR finite as well as gauge independent, if one takes into account the gauge independent renormalization of the mixing matrices as presented in [45] for the Equations (30) and (31). Similar to the case of $\overline{\text{DR}}$ masses an effective neutrino mass matrix of the form

$$(m_{\nu\nu}^{\text{eff}})_{ij} = a\Lambda_i\Lambda_j + b(\epsilon_i\Lambda_j + \epsilon_j\Lambda_i) + c\epsilon_i\epsilon_j \quad (86)$$

is obtained at one-loop level [30, 31].

3.5. Calculation of the neutrino parameters

With the one-loop on-shell masses $m_{\nu_i}^{1L}$ of the neutrinos and the mixing matrix \mathcal{N}^{1L} one can calculate the relevant parameters to be compared with experimental neutrino data:

$$\Delta m_{atm}^2 = (m_{\nu_3}^{1L})^2 - (m_{\nu_1}^{1L})^2, \quad \Delta m_{sol}^2 = (m_{\nu_2}^{1L})^2 - (m_{\nu_1}^{1L})^2 \quad (87)$$

$$\tan^2 \theta_{atm} = \left| \frac{\mathcal{N}_{3,6}^{1L}}{\mathcal{N}_{3,7}^{1L}} \right|^2, \quad \tan^2 \theta_{sol} = \left| \frac{\mathcal{N}_{2,5}^{1L}}{\mathcal{N}_{1,5}^{1L}} \right|^2, \quad U_{e3}^2 = |\mathcal{N}_{3,5}^{1L}|^2 \quad (88)$$

The formulas are valid for BRpV, in case of the $\mu\nu$ SSM one has to replace $\mathcal{N}_{i,j}^{1L} \rightarrow \mathcal{N}_{i,j+1}^{1L}$.

Parameter	Best fit	2σ
$\Delta m_{21}^2 [10^{-5} \text{ eV}^2]$	$7.59^{+0.20}_{-0.18}$	$7.24 - 7.99$
$ \Delta m_{31}^2 [10^{-3} \text{ eV}^2]$	$2.45^{+0.09}_{-0.09}$	$2.28 - 2.64$
$\tan^2 \theta_{12} = \tan^2 \theta_{sol}$	$0.453^{+0.037}_{-0.031}$	$0.39 - 0.54$
$\tan^2 \theta_{23} = \tan^2 \theta_{atm}$	$1.04^{+0.28}_{-0.22}$	$0.69 - 1.56$
$\tan^2 \theta_{13} = \tan^2 \theta_R$	$0.010^{+0.009}_{-0.006}$	≤ 0.028

Table 1: Current bounds on neutrino data taken from [10].

4. One-loop on-shell neutrino masses

In this section we show the results for the one-loop on-shell neutrino masses in BRpV and the $\mu\nu$ SSM, which we obtain using the definitions of the last sections. The two models and the procedure explained before are also added to `CNNDecays` [51], so that the reader is able to reproduce all the presented results.

4.1. Parameter choice

In the choice of our parameters we closely follow the strategy presented in [47]. In Table 1 we show the current bounds on neutrino data taken from [10], which will be used for our later analysis. Moreover we note for the specific models:

- **BRpV:** We use the low-energy parameter sets of the well-known “Snowmass Points and Slopes” (SPS) [61, 62] as benchmark scenarios. In the discussion of the decay widths we refer to two particular scenarios, which are defined in Table 2. SPS 2 was changed such, that a higgsino-like lightest neutralino is present by setting $M_1 = M_2 = 600 \text{ GeV}$. The additional parameters ϵ_i, v_i or respectively ϵ_i, Λ_i are fixed by the neutrino constraints, which can be found in Table 1, if not mentioned to be chosen otherwise. The corresponding B_i are determined from the tadpole equations.
- **$\mu\nu$ SSM:** We refer to the low-energy parameter sets of the NMSSM benchmark scenarios [42, 63] named mSUGRA i or GMSB j based on the soft SUSY breaking mechanism. Details can also be found within [45]. We add for completeness, that the VEV v_c is fixed from μ and λ using Equation (8). In the discussion of the decay widths we use four scenarios, which are defined in Table 2 and which are chosen such, that the lightest neutralino is either bino, wino, higgsino or singlino. Concerning perturbativity and charge or color breaking minima we refer to [47, 64] and note that our choices of λ, κ and T_λ, T_κ respect the given constraints. The additional parameters v_i, Y_ν^i or respectively ϵ_i, Λ_i are fixed by the neutrino constraints, which can be found in Table 1, if not mentioned to be chosen otherwise. The corresponding T_ν^i are determined from the tadpole equations.

	SPS 2'	SPS 3	mSUGRA 1	mSUGRA 3'	mSUGRA 4'	GMSB 5
Changes	$M_1 = M_2$ $= 600 \text{ GeV}$	None	None	$M_1 \leftrightarrow M_2$	$\kappa \approx 0.1$ $\rightarrow 0.4$	None
$\tilde{\chi}_1^0 : m$ m^{1L} C	371.05 370.68 \tilde{H}	165.03 164.76 \tilde{B}	210.79 210.61 \tilde{B}	208.78 208.55 \tilde{W}	196.82 199.51 \tilde{H}	203.30 203.30 \tilde{S}
$\tilde{\chi}_2^0 : m$ m^{1L} C	395.76 395.67 \tilde{H}	295.36 295.73 \tilde{W}	387.18 387.10 \tilde{W}	391.11 390.82 \tilde{B}	205.60 205.24 \tilde{H}	496.87 496.81 \tilde{B}
$\tilde{\chi}_3^0 : m$ m^{1L} C	600.00 599.87 \tilde{B}	504.42 504.76 \tilde{H}	971.11 971.75 \tilde{H}	941.92 940.00 \tilde{H}	327.26 326.83 \tilde{S}	899.60 899.98 \tilde{W}
$\tilde{\chi}_4^0 : m$ m^{1L} C	624.71 624.42 \tilde{W}	521.86 520.49 \tilde{H}	976.52 975.14 \tilde{H}	942.49 943.05 \tilde{H}	330.44 330.95 \tilde{B}	1377.67 1377.45 \tilde{H}
$\tilde{\chi}_5^0 : m$ m^{1L} C			2101.57 2101.57 \tilde{S}	1421.70 1421.66 \tilde{S}	608.43 607.65 \tilde{W}	1383.97 1383.04 \tilde{H}
$\tilde{\chi}_1^\pm : m$ m^{1L} C	377.77 378.22 \tilde{H}^\pm	295.07 295.58 \tilde{W}^\pm	387.16 387.23 \tilde{W}^\pm	208.83 208.77 \tilde{W}^\pm	201.36 201.75 \tilde{H}^\pm	899.59 900.14 \tilde{W}^\pm
$\tilde{\chi}_2^\pm : m$ m^{1L} C	619.84 619.13 \tilde{W}^\pm	521.90 521.03 \tilde{H}^\pm	977.07 976.69 \tilde{H}^\pm	946.04 945.71 \tilde{H}^\pm	608.40 607.81 \tilde{W}^\pm	1384.24 1383.51 \tilde{H}^\pm

Table 2: 2 scenarios for BRpV and 4 scenarios for the $\mu\nu$ SSM to allow for lightest neutralinos with different particle characters C where m and m^{1L} represent the tree level and one-loop masses; SPS 2, mSUGRA 3 and 4 were adopted, so that also a wino- and higgsino-like lightest neutralino is accessible.

4.2. Absolute neutrino masses and mass differences

First we illustrate the behavior of the absolute neutrino masses and point out, that the on-shell renormalization allows a similar parameter dependence as the $\overline{\text{DR}}$ masses, which were defined in [30].

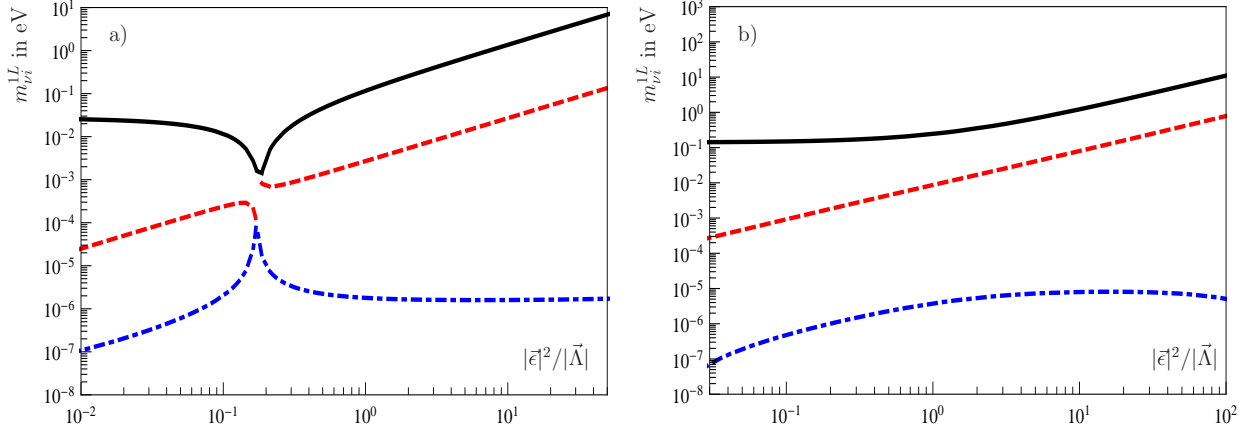


Figure 1: Three on-shell neutrino masses $m_{\nu_i}^{1L}$ in eV as a function of $|\vec{\epsilon}|^2/|\vec{\Lambda}|$ for a scenario in a) the $\mu\nu\text{SSM}$ based on mSUGRA 1; b) BRpV based on SPS 3.

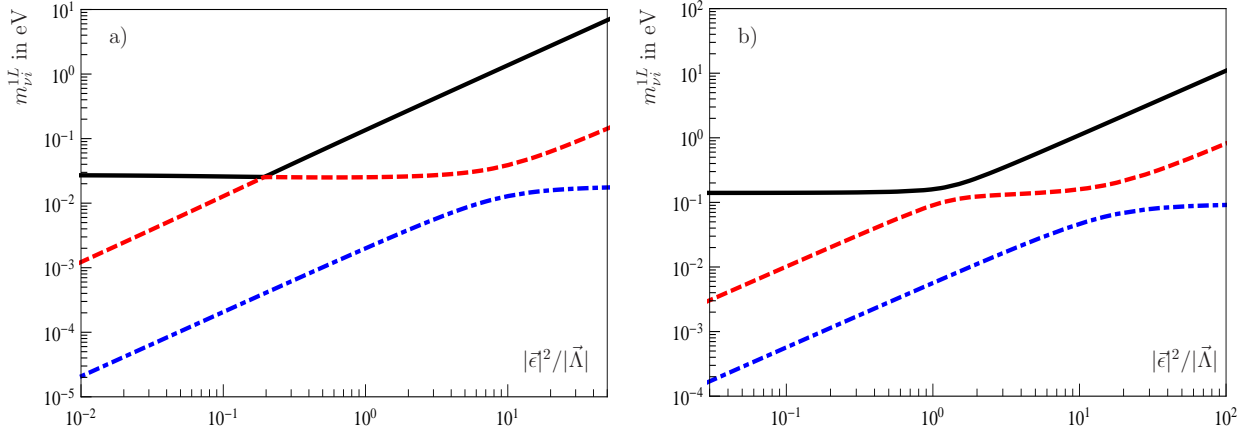


Figure 2: Three on-shell neutrino masses $m_{\nu_i}^{1L}$ in eV as a function of $|\vec{\epsilon}|^2/|\vec{\Lambda}|$ using the sign-condition defined in Equation (89) for a scenario in a) $\mu\nu\text{SSM}$ based on mSUGRA 1; b) BRpV based on SPS 3.

Figure 1 shows the neutrino masses $m_{\nu_i}^{1L}$ of the three left-handed neutrinos as a function of $|\vec{\epsilon}|^2/|\vec{\Lambda}|$, where we have chosen $\epsilon_1 = \epsilon_2 = \epsilon_3$ and $\Lambda_1 = \Lambda_2 = \Lambda_3$ and a fixed value of $|\vec{\Lambda}| = 0.235 \text{ GeV}^2$. In case of the $\mu\nu\text{SSM}$ the scenario is based on mSUGRA 1 and in case of BRpV on SPS 3. In Figure 2 we set $\Lambda_1 = -\Lambda_2 = \Lambda_3$, so that the sign-condition

$$\frac{\epsilon_2}{\epsilon_3} \frac{\Lambda_2}{\Lambda_3} < 0 \quad (89)$$

is fulfilled. This allows for a simpler fit to the solar angle, since it helps to decouple the atmospheric and the solar problem by reducing the contributions from the b -term in the

effective neutrino mass matrix at one-loop level in Equation (86) [30, 31]. We will therefore make use of the sign-condition in the following. Both models show a similar behavior.

The absolute value of $|\vec{\epsilon}|$ determines the neutrino masses $m_{\nu_1}^{1L}$ and $m_{\nu_2}^{1L}$, which are generated at one-loop level, whereas the tree level neutrino mass $m_{\nu_3}^{1L}$ is set constant by $|\vec{\Lambda}|$ for small $|\vec{\epsilon}|$ and is only affected by the one-loop corrections for large values of $|\vec{\epsilon}|$. Note that for the explanation of the neutrino data the individual ϵ_i and Λ_i have to be chosen differently. In case of the scenarios without sign-condition a level-crossing as in Figure 1 a) can take place, which existence is dependent on the parameter point in the $\mu\nu$ SSM as well as in BRpV. It corresponds to a sign-flip between $m_{\nu_1}^{1L}$ and $m_{\nu_3}^{1L}$.

The Yukawa couplings Y_l of the leptons at tree level have to be adopted such, that the tree level lepton masses coincide with the experimental values. Suppose we have fitted the tree level lepton masses m_l to the experimental values, we can define the relative one-loop corrections by

$$\delta_l = \left| \frac{m_l^{1L} - m_l}{m_l} \right| . \quad (90)$$

For the $\mu\nu$ SSM scenarios already presented in Figures 1 a) and 2 a) with respect to the neutrino masses, Figure 3 shows the relative correction δ_l for the lepton masses at one-loop level. For reasonable neutrino masses the corrections to the lepton masses are small $\delta_l < 10^{-10}$, even for the electron below the experimental uncertainties. Note that the shown dips correspond to a sign change, since we show the absolute value of the correction.

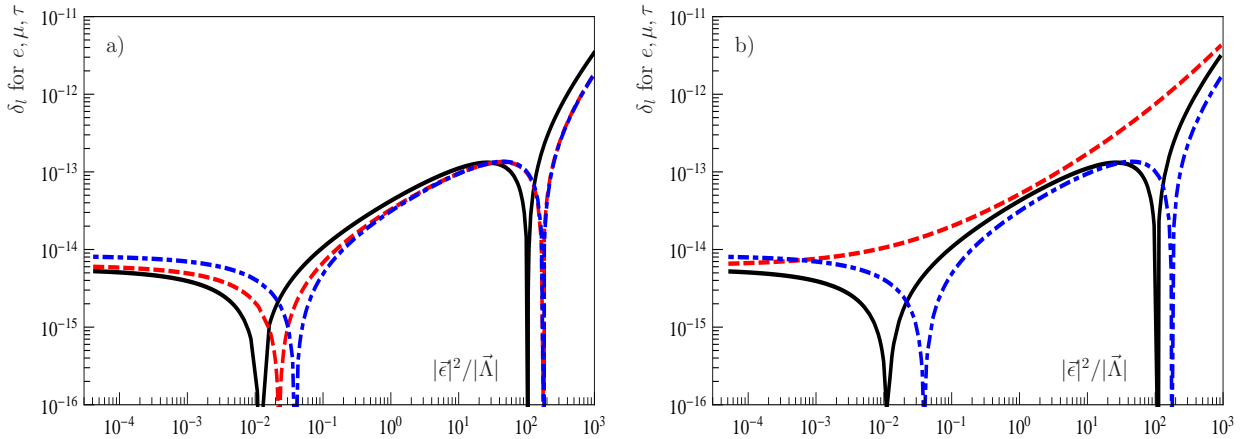


Figure 3: Corrections δ_l defined in Equation (90) for the three lepton masses (τ (black, solid), μ (red, dashed), e (blue, dot-dashed)) as a function of $|\vec{\epsilon}|^2/|\vec{\Lambda}|$ for the $\mu\nu$ SSM as in Figure 1 a), in detail a) without sign-condition (89); b) with sign-condition (89).

In the following we want to point out the difference between the $\overline{\text{DR}}$ masses in [30] and the on-shell masses as defined in this article. For BRpV based on SPS 3 we present both schemes in Figure 4.

Although there exists a difference in the lightest neutrino mass $m_{\nu_1}^{1L}$, the mass differences Δm_{atm}^2 and Δm_{sol}^2 defined in Equation (87) are very similar, since those are given by the

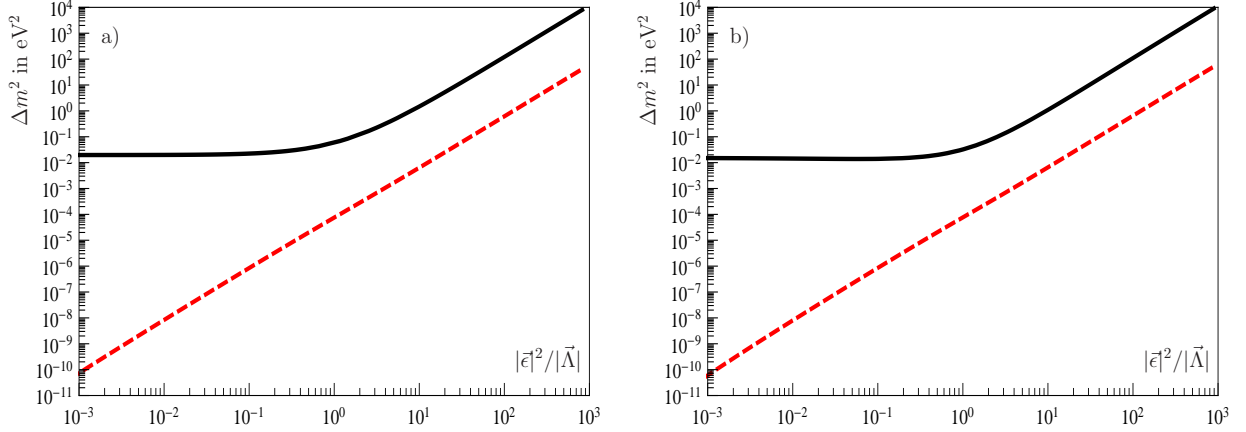


Figure 4: Mass differences Δm_{atm}^2 (black, solid) and Δm_{sol}^2 (red, dashed) as a function of $|\vec{\epsilon}|^2/|\vec{\Lambda}|$ for BRpV based on SPS 3 using the a) on-shell masses defined in Section 3; b) $\overline{\text{DR}}$ masses as defined in [30].

absolute values of $m_{\nu_3}^{1L}$ and $m_{\nu_2}^{1L}$. Since the on-shell procedure tends to reduce the impact of the one-loop contributions, the largest mass $m_{\nu_3}^{1L}$ is in some scenarios only for larger values of $|\vec{\epsilon}|$ affected by the one-loop contributions when comparing the on-shell with the $\overline{\text{DR}}$ masses. However, for the neutrinos the two renormalization prescriptions are very similar, whereas for the other neutralinos and charginos and in particular for the leptons the corrections are (much) smaller in case of on-shell masses compared to the $\overline{\text{DR}}$ masses. In the scenarios above δ_l for $\overline{\text{DR}}$ lepton masses is between 0.3 and 2%. The one-loop corrections to the neutralino and chargino masses are included in Table 2 and are typically in the per-mil range, whereas in case of $\overline{\text{DR}}$ masses the corrections are in the order of a few per-cent [44].

4.3. Relation between $\vec{\Lambda}$, $\vec{\epsilon}$ and the neutrino mass differences/the mixing angles

In this section we discuss the relations between the neutrino mass differences, the mixing angles and the alignment parameters $\vec{\epsilon}$ and $\vec{\Lambda}$. We refer to [10] for the current neutrino data, which we presented in Table 1. For all figures presented in this section we fit $\vec{\epsilon}$ and $\vec{\Lambda}$ such, that the neutrino data bounds are fulfilled except for the angle shown in the figure.

Similar to [30] we use $\vec{\Lambda}$ at tree level to explain the atmospheric mass difference and the atmospheric mixing angle, whereas $\vec{\epsilon}$ respectively $\vec{\tilde{\epsilon}}$ at one-loop level is used for the solar mass difference and the solar mixing angle. The latter correlation to $\vec{\tilde{\epsilon}}$ is more distinct than the one to $\vec{\epsilon}$, where a prerotation with the matrix \mathcal{N}_ν diagonalizing the tree level neutrino mass matrix $m_{\nu\nu}^{\text{eff}}$ given in Equation (24) according to $\vec{\tilde{\epsilon}} = \mathcal{N}_\nu \vec{\epsilon}$ was performed. The mixing matrix \mathcal{N}_ν is in the considered scenarios exactly given by [65]

$$\mathcal{N}_\nu = \begin{pmatrix} \frac{\sqrt{\Lambda_2^2 + \Lambda_3^2}}{|\vec{\Lambda}|} & -\frac{\Lambda_1 \Lambda_2}{\sqrt{\Lambda_2^2 + \Lambda_3^2} |\vec{\Lambda}|} & -\frac{\Lambda_1 \Lambda_3}{\sqrt{\Lambda_2^2 + \Lambda_3^2} |\vec{\Lambda}|} \\ 0 & \frac{\Lambda_3}{\sqrt{\Lambda_2^2 + \Lambda_3^2}} & -\frac{\Lambda_2}{\sqrt{\Lambda_2^2 + \Lambda_3^2}} \\ \frac{\Lambda_1}{|\vec{\Lambda}|} & \frac{\Lambda_2}{|\vec{\Lambda}|} & \frac{\Lambda_3}{|\vec{\Lambda}|} \end{pmatrix}. \quad (91)$$

We will first comment on the atmospheric and solar mass differences Δm_{atm}^2 and Δm_{sol}^2 . Whereas the atmospheric mass difference is determined by $|\vec{\Lambda}|$, the solar mass difference correlates with $|\vec{\epsilon}|$. Therein the absolute value of $\vec{\Lambda}$ can be estimated using Equation (29).

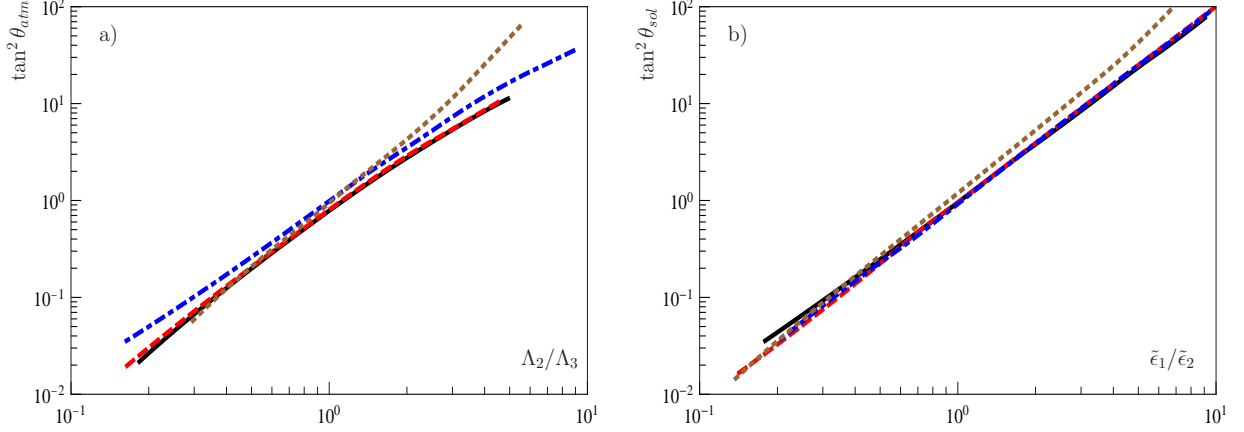


Figure 5: Correlation between the alignment parameters and the neutrino mixing angles for the $\mu\nu$ SSM based on mSUGRA 1 (black), 3' (red, dashed), 4' (blue, dot-dashed) and GMSB 5 (brown, dotted) presented in Table 2, in detail a) $\tan^2 \theta_{atm}$ as a function of Λ_2/Λ_3 ; b) $\tan^2 \theta_{sol}$ as a function of $\tilde{\epsilon}_1/\tilde{\epsilon}_2$.

Using the definitions of Equation (88) for the neutrino mixing angles we will now focus on the correlation between the alignment parameters Λ_i and $\tilde{\epsilon}_i$ and the neutrino mixing angles, which is shown in Figure 5. The ratio Λ_2/Λ_3 defines the atmospheric angle and $\tilde{\epsilon}_1/\tilde{\epsilon}_2$ fits the solar angle. The reactor angle is determined by $\Lambda_1/\sqrt{\Lambda_2^2 + \Lambda_3^2}$ [30] which however can receive sizable loop corrections.

5. Tree level decay widths of $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$

After having discussed the finite mass corrections between tree level and one-loop level, which allow to explain the full neutrino spectrum, we will now turn to the decays $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ in order to study the correlations between neutrino mixing angles and the different leptonic final states. The partial widths for the decays $\tilde{\chi}_j^0 \rightarrow \tilde{\chi}_l^\pm W^\mp$ including the \mathcal{R}_p decays $\tilde{\chi}_1^0 \rightarrow l_l^\pm W^\mp$ are obtained from the following interaction Lagrangian

$$\mathcal{L} = \overline{\tilde{\chi}_l^-} \gamma^\mu (P_L O_{Llj} + P_R O_{Rlj}) \tilde{\chi}_j^0 W_\mu^- + h.c. \quad . \quad (92)$$

The couplings are given by

$$O_{Llj} = -g \mathcal{N}_{j2}^* U_{l1} - \frac{1}{\sqrt{2}} g \left(\mathcal{N}_{j3}^* U_{l2} + \sum_{k=1}^3 U_{l,2+k} \mathcal{N}_{j,4(5)+k}^* \right) \quad (93)$$

$$O_{Rlj} = -g V_{l1}^* \mathcal{N}_{j2} + \frac{1}{\sqrt{2}} g V_{l2}^* \mathcal{N}_{j4} \quad , \quad (94)$$

where in Equation (93) the index is 4 (5) in case of BRpV ($\mu\nu$ SSM). The widths have the form

$$\Gamma^0 = \frac{1}{16\pi m_i^3} \sqrt{\kappa(m_i^2, m_o^2, m_W^2)} \frac{1}{2} \sum_{pol} |M_T|^2 \quad , \quad (95)$$

where m_i (m_o) is the mass of the mother (daughter) particle and M_T is the tree level matrix element. Explicitly they are given by

$$\begin{aligned} \Gamma^0(\tilde{\chi}_j^0 \rightarrow \tilde{\chi}_l^+ W^-) &= \frac{1}{16\pi m_j^3} \sqrt{\kappa(m_j^2, m_l^2, m_W^2)} \\ &\times \left((|O_{Llj}|^2 + |O_{Rlj}|^2) f(m_j^2, m_l^2, m_W^2) - 6\text{Re}(O_{Llj} O_{Rlj}^*) m_j m_l \right) \end{aligned} \quad (96)$$

$$\begin{aligned} \Gamma^0(\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_k^0 W^+) &= \frac{1}{16\pi m_i^3} \sqrt{\kappa(m_i^2, m_k^2, m_W^2)} \\ &\times \left((|O_{Lik}|^2 + |O_{Rik}|^2) f(m_i^2, m_k^2, m_W^2) - 6\text{Re}(O_{Lik} O_{Rik}^*) m_i m_k \right) \end{aligned} \quad (97)$$

using the functions

$$f(x, y, z) = \frac{1}{2}(x + y) - z + \frac{(x - y)^2}{2z} \quad (98)$$

$$\kappa(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \quad . \quad (99)$$

Coupling $\tilde{\chi}_1^0 - l_i^\mp - W^\pm$

The tree level couplings O_{Li1} in Equation (93) and O_{Ri1} in Equation (94) for the special case of the lightest neutralino $\tilde{\chi}_1^0$ coupling to a lepton l_i can be approximated in terms of the alignment parameters Λ_i and ϵ_i , which allows to understand the correlations to the neutrino mixing angles. Although we have shown these approximated couplings in great detail already in [47] for the $\mu\nu$ SSM, their knowledge is of great importance. The right-handed couplings O_{Ri1} can be approximated in BRpV and the $\mu\nu$ SSM by:

$$O_{Ri1} \approx \frac{g(Y_l)_{ii} v_d}{2\text{Det}_+} \left[\frac{g v_u N_{12} - M_2 N_{14}}{\mu} \epsilon_i + \frac{g(2\mu^2 + g^2 v_u^2) N_{12} - g^2(v_d \mu + M_2 v_u) N_{14}}{2\mu \text{Det}_+} \Lambda_i \right] \quad (100)$$

Therein the determinant of the heavy states in the chargino mass matrix appears

$$\text{Det}_+ = M_2 \mu - \frac{1}{2} g^2 v_d v_u \quad . \quad (101)$$

In contrast the left-handed couplings differ in both models and are given by the following expressions:

$$\begin{aligned} O_{Li1}^{bR_p} &\approx \frac{g\Lambda_i}{\sqrt{2}} \left[-\frac{g' M_2 \mu}{2\text{Det}_0^{bR_p}} N_{11} + \left(\frac{g}{\text{Det}_+} + \frac{g M_1 \mu}{2\text{Det}_0^{bR_p}} \right) N_{12} \right. \\ &\quad \left. - \frac{v_u}{2} \left(\frac{m_\gamma}{2\text{Det}_0^{bR_p}} + \frac{g^2}{\mu \text{Det}_+} \right) N_{13} + \frac{v_d m_\gamma}{4\text{Det}_0^{bR_p}} N_{14} \right] \end{aligned} \quad (102)$$

$$\begin{aligned}
O_{Li1}^{\mu\nu\text{SSM}} \approx \frac{g\Lambda_i}{\sqrt{2}} \Bigg[& -\frac{g'M_2}{2\text{Det}_0^{\mu\nu\text{SSM}}}(v_d v_u \lambda^2 + m_R \mu) N_{11} \\
& + \left(\frac{g}{\text{Det}_+} + \frac{gM_1}{2\text{Det}_0^{\mu\nu\text{SSM}}}(v_d v_u \lambda^2 + m_R \mu) \right) N_{12} \\
& - \left(\frac{g^2 v_u}{2\mu \text{Det}_+} - \frac{m_\gamma}{8\mu \text{Det}_0^{\mu\nu\text{SSM}}}(\lambda^2 v_d (v_d^2 + v_u^2) + 2m_R \mu v_u) \right) N_{13} \\
& + \frac{m_\gamma}{8\mu \text{Det}_0^{\mu\nu\text{SSM}}}(\lambda^2 v_u (v_d^2 + v_u^2) + 2m_R \mu v_d) N_{14} \\
& - \frac{\lambda m_\gamma}{4\sqrt{2}\text{Det}_0^{\mu\nu\text{SSM}}}(v_u^2 - v_d^2) N_{15} \Bigg] \tag{103}
\end{aligned}$$

The couplings include the abbreviations of Equations (8),(9) and (26) and the determinants shown in Equations (27) and (28). Neglecting the right-handed couplings, which are generally smaller than the left-handed ones, it is obvious that the lightest neutralino $\tilde{\chi}_1^0$ couples to $l_i^\pm W^\mp$ proportional to Λ_i without any dependence on the ϵ_i parameters. This fact is in case of the considered BRpV and the $\mu\nu\text{SSM}$ independent of the nature of the lightest neutralino $\tilde{\chi}_1^0$. However, it is important to emphasize that all previous formulas are tree level results. A priori it is not clear, what happens if one uses the mixing matrices \mathcal{N} , U and V for masses at one-loop level or performs a complete NLO calculation of the decay width.

6. NLO decay widths of $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ and correlations to neutrino mixing angles

In this section we focus on the NLO decay widths $\Gamma^1(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp)$. The calculation procedure was introduced in [45] and can be adopted for BRpV and the $\mu\nu\text{SSM}$ without any changes. The corresponding formulas have been implemented in the public program `CNNDecays` [51]. All technical questions concerning the used renormalization scheme like the cancellation of UV and IR divergences or gauge invariance have been discussed in [45] for the (N)MSSM and preserve their validity for the R -parity violating extensions.

6.1. Absolute size of the corrections

In Figure 6 we present the NLO decay width for a mSUGRA 4 based scenario, where we varied $\kappa = 0.1 - 0.5$ in order to have a singlino- and a higgsino-like lightest neutralino $\tilde{\chi}_1^0$ in comparison. For $\kappa \lesssim 0.1$ the decay is near the kinematical threshold and for $\kappa \gg 0.5$ a Landau pole appears. We show the relative correction $\delta(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp)$, which is defined by:

$$\delta(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp) = \frac{\Gamma^1(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp) - \Gamma^0(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp)}{\Gamma^0(\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp)} \tag{104}$$

The size and the sign of the corrections is strongly dependent on the size and the sign of the R -parity violating parameters v_i and Y_ν^i or Λ_i and ϵ_i . In Figure 6 we set $\vec{\Lambda} = (0.31, 5.21, 2.02) \cdot 10^{-2} \text{ GeV}^2$ and $\vec{\epsilon} = (7.49, 9.61, -6.57) \cdot 10^{-3} \text{ GeV}$. Since the solar mass difference and mixing angle are only well defined at one-loop level, the corrections to the decay $\tilde{\chi}_1^0 \rightarrow e^\pm W^\mp$ are sizable and can be of the order of the tree level decay width or larger. The corrections to $\tilde{\chi}_1^0 \rightarrow \mu^\pm W^\mp$ or $\tau^\pm W^\mp$ are generally smaller, but remain of order 10%.

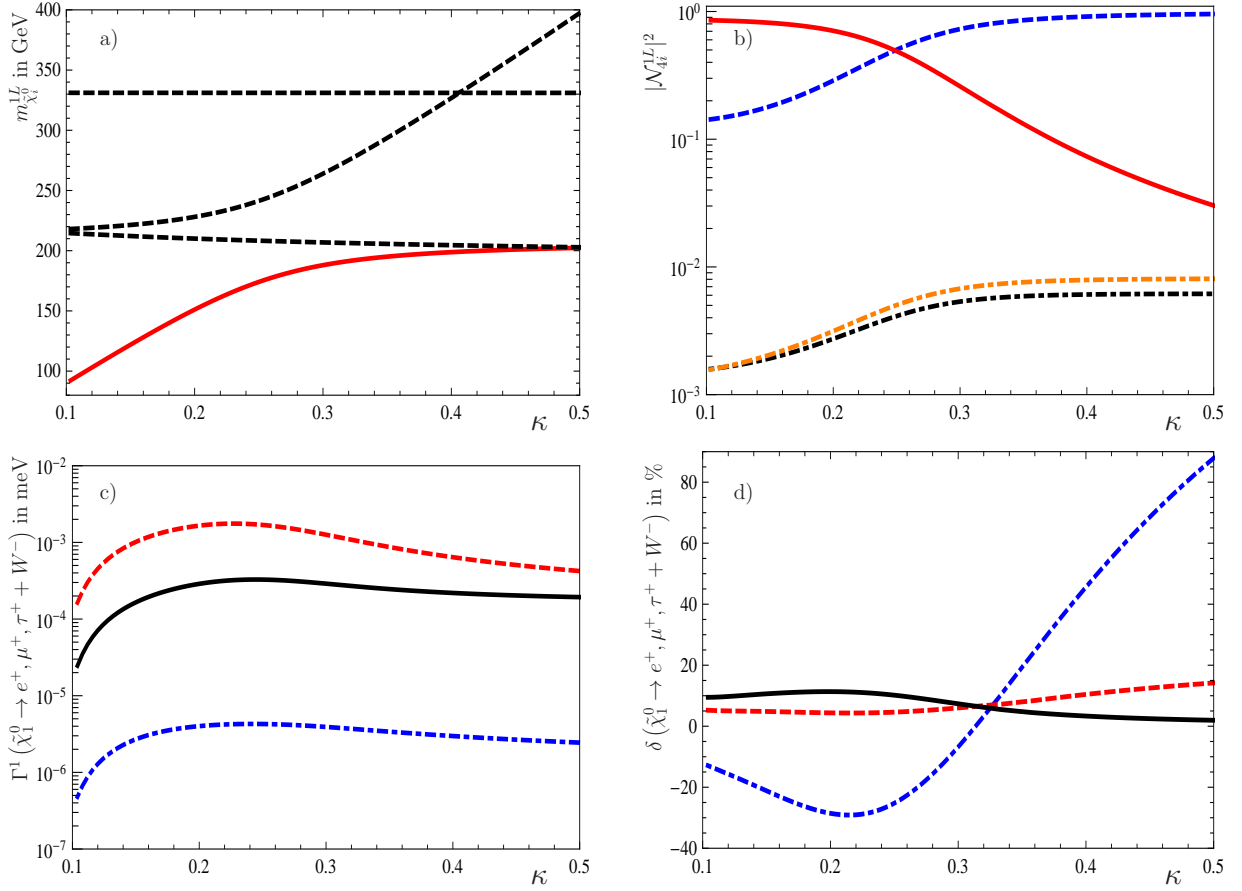


Figure 6: Neutralino masses, particle content of the lightest neutralino and decay widths $\Gamma^1(\tilde{\chi}_1^0 \rightarrow l^+ W^-)$ as a function of κ within the $\mu\nu$ SSM based on the mSUGRA 4 scenario and $\vec{\Lambda} = (0.31, 5.21, 2.02) \cdot 10^{-2} \text{ GeV}^2$ and $\vec{\epsilon} = (7.49, 9.61, -6.57) \cdot 10^{-3} \text{ GeV}$: a) Neutralino masses $m_{\tilde{\chi}_i^0}^{1L}$ ($m_{\tilde{\chi}_1^0}^{1L}$ (red, solid), $m_{\tilde{\chi}_{2,3,4}^0}^{1L}$ (black, dashed)); b) Particle character $|\mathcal{N}_{4i}^{1L}|^2$ ($\tilde{\nu}^c$ (red, solid), $\tilde{H}_u + \tilde{H}_d$ (blue, dashed), \tilde{B} (black, dot-dashed), \tilde{W} (orange, dot-dashed)); c) NLO decay width Γ^1 (e (blue, dot-dashed), μ (red, dashed), τ (black)); d) Relative correction δ defined in Equation (104) (e (blue, dot-dashed), μ (red, dashed), τ (black)).

6.2. Correlation between decay widths and neutrino mixing angles

Having discussed the absolute size of the NLO contributions to the decay widths of the two body decays $\tilde{\chi}_1^0 \rightarrow l^+ W^-$, we will now focus on the correlations between the branching ratios and the neutrino mixing angles. We have shown in Section 5, that the tree level couplings can be approximated in terms of the alignment parameters $\vec{\Lambda}$ and $\vec{\epsilon}$. Since the alignment parameters are connected to the neutrino mixing angles as explained in Section 4.3, one can expect the following tree level relation:

$$\frac{\Gamma^0(\tilde{\chi}_1^0 \rightarrow \mu^+ W^-)}{\Gamma^0(\tilde{\chi}_1^0 \rightarrow \tau^+ W^-)} \propto \left(\frac{O_{L21}}{O_{L31}} \right)^2 \approx \left(\frac{\Lambda_2}{\Lambda_3} \right)^2 \approx \tan^2 \theta_{atm} \quad (105)$$

It is a priori not clear to which extent this relation also holds at one-loop level. We have found that using one-loop corrected $\overline{\text{DR}}$ masses together with the corresponding one-loop

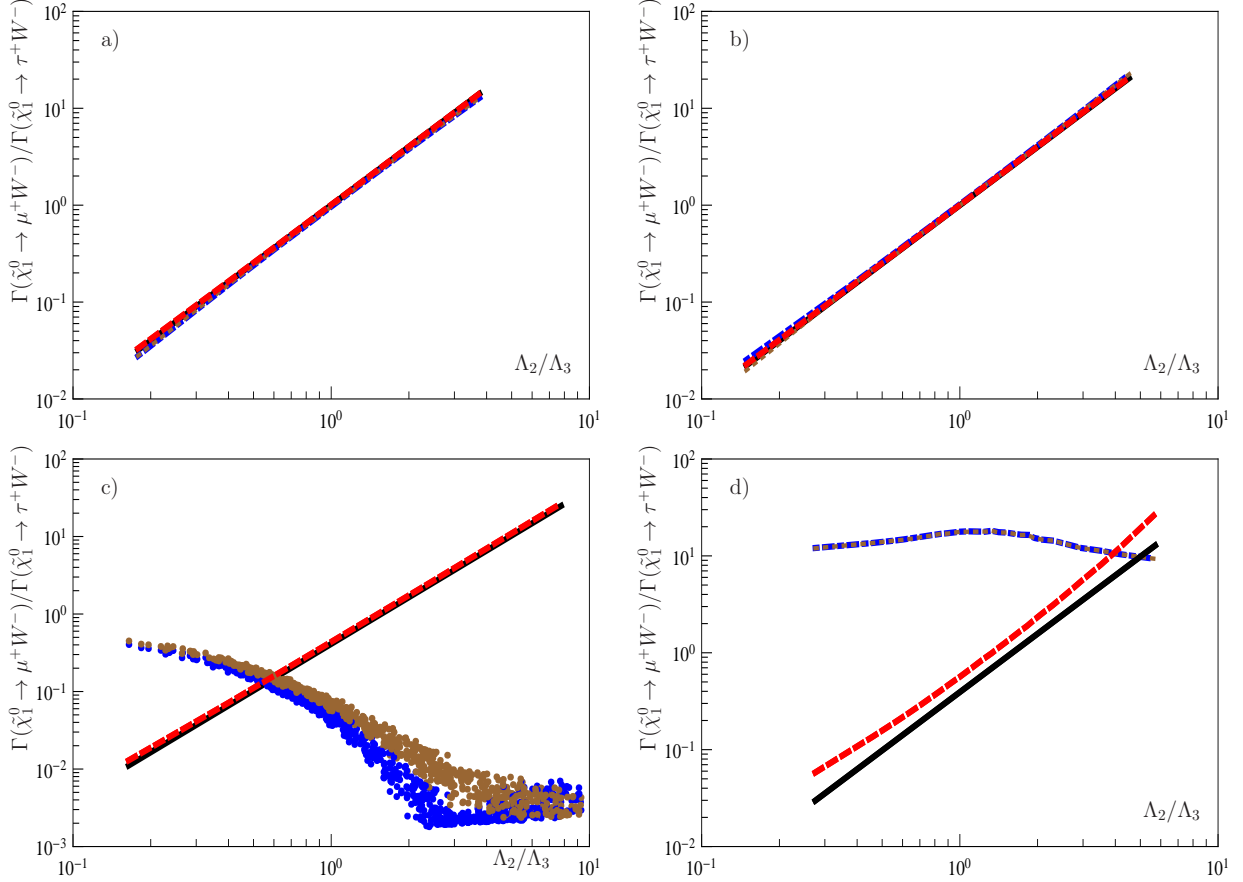


Figure 7: Ratios $\Gamma(\tilde{\chi}_1^0 \rightarrow \mu^+ W^-)/\Gamma(\tilde{\chi}_1^0 \rightarrow \tau^+ W^-)$ with $\Gamma = \Gamma^0$ in black, $\Gamma = \Gamma^1$ in red, dashed, $\Gamma = \Gamma^0$ with \mathcal{N}^{1L} in blue, dot-dashed, $\Gamma = \Gamma^0$ with \mathcal{N}^{1L}, U^{1L} and V^{1L} in brown, dotted as a function of Λ_2/Λ_3 for the $\mu\nu$ SSM scenarios of Table 2: a) mSUGRA 1; b) mSUGRA 3'; c) mSUGRA 4'; d) GMSB 5.

corrected mixing matrix \mathcal{N}^{1L} in the tree level decay width can spoil this relation. However, performing the full one-loop corrections including the real corrections from photon emission in the considered on-shell scheme retains the predictions at tree level, so that

$$\frac{\Gamma^1(\tilde{\chi}_1^0 \rightarrow \mu^+ W^-)}{\Gamma^1(\tilde{\chi}_1^0 \rightarrow \tau^+ W^-)} \propto \left(\frac{\Lambda_2}{\Lambda_3}\right)^2 \approx \tan^2 \theta_{atm} \quad . \quad (106)$$

In Figure 7 we highlight this feature using the scenarios of Table 2 for the $\mu\nu$ SSM. We do not only show the tree level and one-loop relations, but also the relations obtained by using the one-loop mixing matrices \mathcal{N}^{1L}, U^{1L} and V^{1L} within the tree level decay width $\Gamma^0(\tilde{\chi}_1^0 \rightarrow l^+ W^-)$, see Equation (96). The ratios are shown as a function of Λ_2/Λ_3 , which is equivalent to varying $\tan^2 \theta_{atm}$ as can be seen in Figure 5. We vary the other parameters such, that the neutrino data are within their 2σ bounds as given in Table 1. This is the cause of the observed widths in case of the mSUGRA 4' scenario.

As already known from BRpV the approximated one-loop contributions using Γ^0 together with the one-loop corrected mixing matrices show the expected behavior if the lightest

neutralino is either mainly a bino, wino or a pure higgsino. However, in case that the singlino-component gets sizable or dominant, these approximations fail and the complete one-loop decay width Γ^1 has to be calculated to obtain a reliable result. In case of mSUGRA 4', which is shown in Figure 7, the higgsino has a purity of 91.7%, but a singlino component of 6.9% is left, so that also in this case the full one-loop decay width is advisable.

The reason for the misleading ratios in case of a singlino-like neutralino is a cancellation between the second and the last terms of the left-handed coupling O_{L1} in Equation (93) at tree level. Replacing the tree level mixing matrix \mathcal{N} by the one-loop mixing matrix \mathcal{N}^{1L} spoils this cancellation, so that unreasonable results for the decay widths and also the ratios are obtained. However, the very same effect is restored once the complete one-loop corrections are included.

For completeness let us briefly discuss the situation within BRpV. As examples we take scenarios where the lightest neutralino is either mainly a bino, scenario SPS 3, or mainly a higgsino, scenario SPS 2'. The ratios of decay widths using the various approximations are shown in Figure 8. We observe that NLO corrections are somewhat more important for higgsino-like neutralinos: Here the combination of tree level decay widths with NLO masses and mixing matrices can lead to correlations which are off by a factor 2 compared the case of the complete NLO calculation. Moreover, the complete NLO corrections can shift the correlations up to 30 per-cent compared to the tree level result.

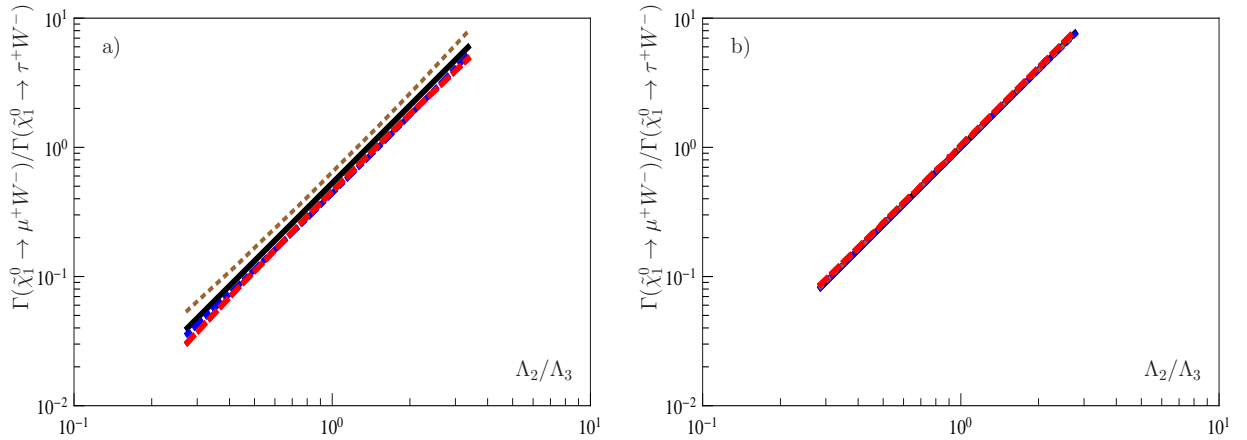


Figure 8: Ratios $\Gamma(\tilde{\chi}_1^0 \rightarrow \mu^+ W^-)/\Gamma(\tilde{\chi}_1^0 \rightarrow \tau^+ W^-)$ with $\Gamma = \Gamma^0$ in black, $\Gamma = \Gamma^1$ in red, dashed, $\Gamma = \Gamma^0$ with \mathcal{N}^{1L} in blue, dot-dashed, $\Gamma = \Gamma^0$ with \mathcal{N}^{1L} , U^{1L} and V^{1L} in brown, dotted as a function of Λ_2/Λ_3 for the BRpV scenarios of Table 2: a) SPS 2'; b) SPS 3.

7. Conclusion

In \mathcal{R}_p models, where the sneutrinos obtain a VEV, a mixing between charged leptons and charginos as well as a mixing between neutrinos and neutralinos occur at tree level. The latter can be used to obtain at least one neutrino mass at tree level. The other neutrino masses can be generated at the one-loop level as it happens e.g. in bilinear \mathcal{R}_p or in the $\mu\nu$ SSM with one generation of right-handed neutrinos. In previous work a $\overline{\text{DR}}$ scheme had been used for this. In this paper we worked out an on-shell scheme and compared both schemes. The challenge here is, that there are less parameters than masses implying finite shifts at the one-loop level to the masses. With respect to the masses of neutrinos and charged leptons we have found only small differences well below the current and near future experimental accuracies.

In both models one expects correlations between ratios of the decay widths or equivalently the branching ratios for the decays $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$ and the atmospheric mixing angle which can be shown easily at tree level using approximate formulas for the mixing matrices. We have calculated these decays including the complete NLO order corrections in the on-shell scheme and compared the results to the pure tree level as well as to the case where one uses tree level formulas for the decay widths but one-loop corrected masses and mixing matrices.

We have found that corrections to the channels containing either a μ or a τ lepton are typically of order ten per-cent whereas the corrections for the e -channel can be of the order of the tree level decay width. The reason for the latter finding is that the solar mixing angle is not well defined at tree level but only at the one-loop level. For the correlations between the decay widths and the atmospheric mixing angle we find that in case of gaugino-like neutralino in all approximations nearly the same correlations. However, already for doublet-higgsino-like neutralino the correlations can be off by a factor two if one combines tree level widths with one-loop corrected masses and mixing angles. In case of a neutralino which has either a sizable singlino-component or which is dominantly singlino-like this is even more pronounced and the correlations get completely wrong in this approximation. Here one has to use the complete NLO corrections to obtain a reliable prediction. Last but not least we note that both models have been implemented in the publicly available program `CNNDecays` for the numerical evaluation.

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Appendix A. New blocks in `CNNDecays`

Both considered models, BRpV and the $\mu\nu$ SSM with one right-handed neutrino superfield, are now supported by `CNNDecays`, which was published in [45] and can be found online [51]. We provide input files for the MSSM, the NMSSM, BRpV and the $\mu\nu$ SSM with one

right-handed neutrino superfield in the folder `/examples`, where in addition the corresponding output files can be found. In BRpV ϵ_i and v_i can be chosen beside the input parameters of the MSSM:

```
Block MODSEL          # Select model
  1      0             # bilinear model
  4      1             # RPviolation
.....
BLOCK RVKAPPAIN
  1      1.45660382E-02 # kappa_1 = eps_1
  2      9.01765562E-03 # kappa_2 = eps_2
  3     -3.16131217E-03 # kappa_3 = eps_3
BLOCK RVSNEVIN
  1     -8.68089903E-04 # v_L_1
  2     -4.50162251E-04 # v_L_2
  3      4.19592513E-04 # v_L_3
.....
```

The input file of the $\mu\nu$ SSM follows the convention of the NMSSM, implying that the VEV of the right-handed sneutrino v_c is calculated from the effective $\mu = \frac{1}{\sqrt{2}}\lambda v_c$. In addition κ and the soft-breaking terms A_λ and A_κ are input. The R -parity violating parameters should be provided via the variables Y_ν^i and v_i :

```
Block MODSEL          # Select model
  3      6             # muonSSM
  4      1             # RPviolation
.....
BLOCK EXTPAR
.....
  61      1.00000000E-01 # lambda
  62      1.08485437E-01 # kappa/2
  63     -9.59966990E+02 # T_lambda/lambda = A_lambda
  64     -1.58051889E+00 # T_kappa/kappa = A_kappa
  65      9.68523016E+02 # mu_eff
  73      5.71920838E-06 # Y_nu_1
  74      6.20206893E-06 # Y_nu_2
  75     -6.20206893E-06 # Y_nu_3
BLOCK RVSNEVIN
  1     -1.35445088E-03 # v_L_1
  2     -1.27271724E-03 # v_L_2
  3      1.71364110E-03 # v_L_3
.....
```

The corresponding soft-breaking terms of Y_ν^i , namely T_ν^i , are calculated from the tadpole equations in addition to $m_{H_d}^2$, $m_{H_u}^2$ and $m_{\tilde{\nu}^c}^2$. The output does not only contain the R -parity violating decays of the lightest neutralino $\tilde{\chi}_1^0 \rightarrow l^\pm W^\mp$, but in addition the relevant parameters for neutrino physics:

```
.....
DECAYTREE  1000022      2.03988581E-13 # chi01
# BR      NDA  ID1  ID2
  5.00974421E-01      2      15      -24 # BR(chi01 -> tau+ W-)
  4.94172793E-01      2      13      -24 # BR(chi01 -> mu + W-)
  4.85278632E-03      2      11      -24 # BR(chi01 -> e+ W-)
DECAY  1000022      2.17956335E-13 # chi01 on NLO
# BR      NDA  ID1  ID2
  4.99329442E-01      2      15      -24 # BR(chi01 -> tau+ W-)
  4.95513455E-01      2      13      -24 # BR(chi01 -> mu + W-)
  5.15710289E-03      2      11      -24 # BR(chi01 -> e+ W-)
.....
```

Block	SPhenoRP	# additional RP parameters
1	5.53918495E-02	# eps_1
2	6.00684651E-02	# eps_2
3	-6.00684651E-02	# eps_3
4	2.10857140E-02	# Lambda_1 = v_d epsilon_1 + mu v_L1 [GeV^2]
5	2.12780743E-01	# Lambda_2 = v_d epsilon_2 + mu v_L2 [GeV^2]
6	2.14264164E-01	# Lambda_3 = v_d epsilon_3 + mu v_L3 [GeV^2]
7	2.24691592E-03	# m^2_atm [eV^2]
8	7.74812242E-05	# m^2_sol [eV^2]
9	9.27311984E-01	# tan^2 theta_atm
10	4.08412270E-01	# tan^2 theta_sol
11	3.54098593E-03	# U_e3^2
15	2.40631533E+01	# v_d
16	2.40631533E+02	# v_u
17	-1.35445088E-03	# v_L1
18	-1.27271724E-03	# v_L2
19	1.71364110E-03	# v_L3

In case of the $\mu\nu$ SSM the given ϵ_i are calculated from Equation (8). The screen output moreover contains not only the one-loop on-shell neutralino and chargino masses, but also the tree level masses for comparison. Please note that for the correct calculation of the one-loop on-shell lepton masses a high accuracy of more than 20 digits is needed to obtain numerical stable results. In most cases for neutrino masses and the decay widths 15 digits are sufficient, as long as all parameters are chosen real, implying that the diagonalization process isn't based on the squared mass matrices. To allow for a higher accuracy, adopt `Selected_real_kind(15, 300)` to `(25, 300)` in `/sphenooriginal/Control.F90` and compile `CNNDecays`. We note that a higher accuracy can result in significantly longer computing times.

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